Asymmetrical Relaying in Molecular Communications

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***Abstract*—Molecular communication via diffusion (MCvD) is a novel communication technique that uses the diffusive characteristics of molecules for enabling the communication between nanomachines. Since the molecules propagate following a random motion, MCvD schemes are usually limited to a short communication range. Most of the molecular relaying schemes in the literature consider symmetric setups where transmitters and receivers are placed at the same distance from the relay, which is difficult to provide in practical scenarios and a possible cause of failure. In this study, asymmetric molecular links of a relay system are investigated. In order to achieve a satisfactory overall performance in spite of the asymmetries, two parameter optimization methods are proposed for the uplink of a relaying system, based on emitting different types of molecules with different diffusion coefficient values from the transmitters. Due to the channel symmetry, the solutions presented in this study are expected to hold for the downlink as well. The resulting bit error rate (BER) performances are presented and discussed.**

***Index Terms*—Molecular communications, nanonetworks, relaying, diffusion coefficient.**

I. Introduction

Molecular communications (MC), inspired by biological systems, has established the idea of structured communication in which the molecule-based signals convey information between nanomachines [1]. MC offers the communications infrastructure for novel applications, especially in the field of medicine. For instance, nano-sensors can be implemented for identifying and even preventing diseases, such as cardiovascular and tumorous disorders [2]. In addition, MC has provided numerous means for the implementation of drug delivery systems [3].

Among several methods proposed in the literature, MC via diffusion (MCvD) is known for being an energy-efficient solution, as the diffusive properties of molecules are utilized for their mobility [4]. In MCvD systems, the information is typically encoded in the characteristics of molecules such as their amount [5], type [6], release or arrival time [7], or other properties. Consequently, there exist different modulation techniques, including on-off keying (OOK) [8], concentration shift keying (CSK) [9], molecular shift keying (MoSK) [10], etc. Due to the randomness that characterizes MCvD systems, they are prone to inter-symbol interference (ISI) [11] and inter-link interference (ILI) [12], impediments that deteriorate the communication performance. As a result, there are vast approaches towards combating ISI and/or ILI. For instance, the authors of [6] propose molecular index-modulation (IM), using the transmit antenna indices for encoding, which is observed to combat ISI and ILI, providing reliable error rates.

Additionally, increasing the distance between the transmitter and receiver leads to a higher probability of decoding error, thus a limited communication range characterizes MCvD systems. In order to mitigate this problem, molecular relaying has been proposed in the literature [13]. The principle of relaying facilitates the communication between two different nanomachines with the help of a third one, through different approaches. For instance, the authors of [14] analyze the performance of an MCvD system, in which the relay decodes and amplifies the signal before forwarding it, thus extending the range of communication. In [15], the authors propose a physical layer network coding scheme that utilizes the reaction of molecules between each other. An estimate-and-forward scheme is proposed in [16], in which maximum likelihood estimation is applied. Similar research studies are proposed and investigated in [17]-[19].

A number of studies in the literature have focused on the optimization of the relay location. For instance, in [20], three nanomachines are located on a straight line and it is found that the best performance is obtained when the relay is placed in the middle of the system. A similar conclusion is drawn in [21]. Meanwhile, the authors of [22] investigate the scenario when three nanomachines are not located on a straight line and the effects of the placement angle are discussed. An optimization problem on the matter is proposed in [23], considering a cooperative communication system.

The majority of the works in the literature consider symmetric scenarios. Nonetheless, practical scenarios might be prone to asymmetries, resulting in different error performances among the communication links established through the relay. As the main purpose of the relay is to enable communication between distant transmitters, such systems are exposed to error propagation. In other words, unequal error protection of the molecular communication links leads to a reduced overall performance. In order to alleviate this issue, we propose an asymmetric scenario in which the two communicating

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nanomachines emit molecules with different diffusion coefficients. The main idea is to optimize the system's parameters in such a way that equal error protection is achieved for the two independent transmitters. In particular, we focus on the quality of the communication links from the transmitter to the relay. Since the diffusive channel characteristics are the same for the uplink and the downlink, the results are expected to hold for the latter as well.

In the literature, the positions of the nanomachines are widely assumed to be fixed. There are also several studies focusing on the impact of the mobility of the nanomachines on the performance of the system, such as [24] and [25]. However, our work also assumes that the transmitters and the relay nodes are stationary.

The main contributions of this study are the design of a novel asymmetrical relay system, parameter optimization of this setup, and performance analysis of the proposed solutions. Moreover, this study could be further introduced to IM schemes, such that the information is encoded on the different diffusion coefficient values.

II. System Model

The basic single-input single-output (SISO) communication link is modeled by a point transmitter and a spherical receiver, placed at a distance r_0 from the point transmitter. The transmitter emits molecules that follow the Brownian motion in their way towards the receiver, which has a radius denoted by r_r, and is assumed to be fully absorbing. The Brownian motion in the 3-D environment can be modeled by a random position change (Δx, Δy, Δz), with a normal distribution $\mathcal{N}(0, 2Ddt)$ of zero mean and $2Ddt$ variance, where dt and D denote the time step and the diffusion coefficient, respectively [26]. The position of each molecule for each time step is calculated as

$$\begin{aligned} x(t_n) &= x(t_{n-1}) + \Delta x, \\ y(t_n) &= y(t_{n-1}) + \Delta y, \\ z(t_n) &= z(t_{n-1}) + \Delta z, \end{aligned} \tag{1}$$

where $(x(t_n), y(t_n), z(t_n))$ denotes the present coordinates, $(x(t_{n-1}), y(t_{n-1}), z(t_{n-1}))$ denotes the previous coordinates and $(\Delta x, \Delta y, \Delta z)$ stands for the position displacement. The fraction of molecules received by time t is modeled by the following equation, as given in [27], known also as the cumulative distribution function (CDF) of the fraction of molecules arriving at the receiver

$$F_{hit}(r_r, r_0, t) = \frac{r_r}{r_0}\text{erfc}\left(\frac{r_0 - r_r}{\sqrt{4\,D\,t}}\right), \tag{2}$$

where $\text{erfc}(\cdot)$ is the complementary error function. As previously mentioned, most of the relay scenarios in the literature assume symmetric communication links. However, this is not necessarily the case in practical applications, as the nanomachines might be located at different distances from the relay node. The proposed scenario is illustrated in Fig. 1, where the orange sphere demonstrates the relay node and the independent point transmitters are shown in blue. r_{01} and r_{02}, the distances from the two transmitters respectively, have different values, and the same symbol time (T_s) is utilized for both transmitters

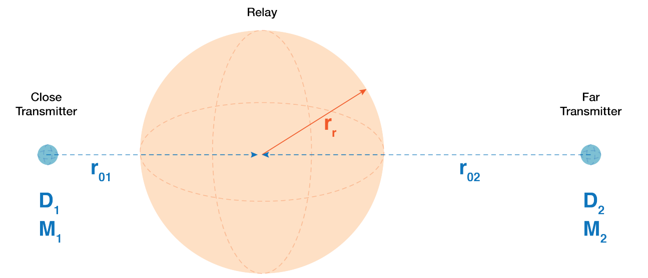

Fig. 1. Asymmetric communication links of a relaying system, where the independent point transmitters are shown in blue and the relay node is the orange sphere.

in order to keep the complexity at a moderate level and achieve synchronous communications. The closer transmitter has an advantage compared to the further one, because the molecules that it emits have to travel a shorter distance to reach the receiver. In order to achieve an overall good quality and fair communication, two parameter optimization methods are proposed for the uplink in the following section. These methods aim to improve the quality of the further transmitter's communication link.

III. Parameter Optimization for Asymmetric Molecular Links

As discussed in the earlier sections, relay schemes are exposed to error propagation, so equal error protection is required for increasing communication reliability of the relay schemes. In order to achieve this for asymmetrical links, the parameters of the two transmitters should be optimized such that similar performances are achieved for the transmitters. We propose to achieve this by minimizing the difference between the received signals from the two transmitters given from the following formulations:

$$F_{hit_1}(r_r, r_{01}) = M_1 \frac{r_r}{r_{01}}\text{erfc}\left(\frac{r_{01} - r_r}{\sqrt{4\,D_1\,T_s}}\right) \tag{3}$$

$$F_{hit_2}(r_r, r_{02}) = M_2 \frac{r_r}{r_{02}}\text{erfc}\left(\frac{r_{02} - r_r}{\sqrt{4\,D_2\,T_s}}\right) \tag{4}$$

This proposed optimization is a diffusion coefficient based approach. Additionally, in order to achieve fairness, it is possible to allow the transmitter placed at a longer distance to emit a higher number of molecules compared to the other one. Given these suggestions, two different optimization algorithms for the uplink of the relaying system are proposed; one varying the diffusion coefficient D_2 only, and the other one varying D_2 and M_2 simultaneously.

A. Optimization problem considering D_2 only

Since the squared error is an amenable technique used in optimization problems, the first presented approach aims to minimize the squared error between the two received signals represented in (3) and (4), given as

$$\epsilon = \left(M_1 \frac{r_r}{r_{01}}\text{erfc}\left(\frac{r_{01} - r_r}{\sqrt{4\,D_1\,T_s}}\right) - M_2 \frac{r_r}{r_{02}}\text{erfc}\left(\frac{r_{02} - r_r}{\sqrt{4\,D_2\,T_s}}\right)\right)^2. \tag{5}$$

The first derivative of (5) with respect to D_2 is taken and the equation is solved for D_2. Assuming that r_r, r_{01}, r_{02}, T_s and D_1 are known values, and considering $M_1 = M_2$, D_2 can be found as

$$D_2 = \frac{\left(\frac{r_{02}-r_r}{\mathrm{erfc}^{-1}\left(\frac{r_{02}}{r_{01}}\mathrm{erfc}\left(\frac{r_{01}-r_r}{\sqrt{4\,D_1\,T_s}}\right)\right)}\right)^2}{4\,T_s}. \tag{6}$$

B. Optimization problem considering both D_2 and M_2

Next, this work presents the second optimization problem that accounts for both D_2 and M_2 parameters. In order to find the values of the unknown parameters for which (5) reaches its minimum value, Newton's algorithm for unconstrained optimization is utilized [28], as given in Algorithm 1.

Algorithm 1 Optimizing D_2 and M_2.

Input: r_r, r_{01}, r_{02}, T_s. D_1, M_1, $error\_tolerance$
Output: M_2, D_2

f: squared error of (4) and (5) as a function of D_2, M_2
$\epsilon = f(\mathrm{D}_2, M_2)$
Define K as

$$K = \frac{\partial^2 f}{\partial M_2^2}\frac{\partial^2 f}{\partial D_2^2} - \left(\frac{\partial^2 f}{\partial D_2 \partial M_2}\right)^2$$

while $\epsilon > error\_tolerance$ **do**

$$(M_2)_{n+1} = (M_2)_n - \frac{|A|}{K((M_2)_n, (D_2)_n)}$$

$$(D_2)_{n+1} = (D_2)_n - \frac{|B|}{K((M_2)_n, (D_2)_n)}$$

Update ϵ
end while

In Algorithm 1, $|A|$ and $|B|$ denote the determinants of the matrices

$$A = \begin{pmatrix} \frac{\partial^2 f}{\partial D_2^2}((D_2)_n, (M_2)_n) & \frac{\partial^2 f}{\partial M_2 \partial D_2}((D_2)_n, (M_2)_n) \\ \frac{\partial f}{\partial D_2}((D_2)_n, (M_2)_n) & \frac{\partial f}{\partial M_2}((D_2)_n, (M_2)_n) \end{pmatrix}$$

$$B = \begin{pmatrix} \frac{\partial^2 f}{\partial M_2^2}((D_2)_n, (M_2)_n) & \frac{\partial^2 f}{\partial M_2 \partial D_2}((D_2)_n, (M_2)_n) \\ \frac{\partial f}{\partial M_2}((D_2)_n, (M_2)_n) & \frac{\partial f}{\partial D_2}((D_2)_n, (M_2)_n) \end{pmatrix}$$

An adequate initialization of M_2 and D_2, by taking into consideration the overall parameters of the system, can result in a faster convergence towards the optimized values.

IV. Simulation Results

Firstly, the case when both transmitters emit molecules with the same D value is considered, in order to compare it with the optimized results afterwards. As a starting point, r_{01}, r_{02}, and r_r are selected to be $8.5\mu m$, $10\mu m$, and $5\mu m$, respectively. The transmitters are independent and they emit 10^6 molecules with coefficient value of $39.7\mu m^2/s$ for bit-1, and nothing for bit-0, as OOK modulation is used. The symbol time is selected to be $0.22s$, and the channel memory is 10 taps. At the relay side, demodulation is performed by simple thresholding for each time slot such that the relay decides on bit-1 if it receives more molecules than the threshold, and bit-0, otherwise. The Poisson-distributed environmental noise is considered to be independent of the source signal, with a mean of $\lambda = 2$, as shown in [29]. A total of 10^6 bit transmissions are simulated for the BER calculations, and the number of the received molecules is modeled following the Gaussian approximation [30]. The BER performances are obtained using computer simulations.

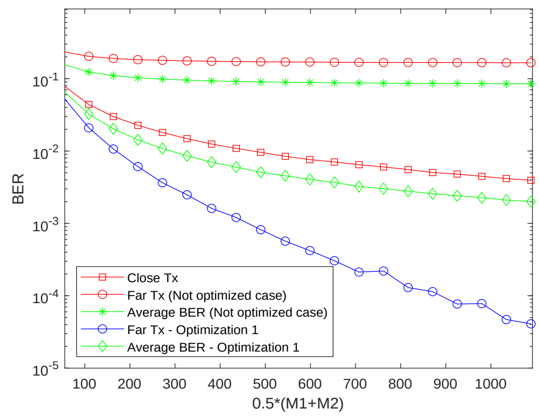

Fig. 2. BER performances of the two communication links for not optimized parameters and the optimized D_2 case.

Secondly, considering all the aforementioned parameters unchanged, the value of D_2 is found as $110.54\mu m^2/s$ using (6). The BER curves presented in Fig. 2 are obtained from the two preceding cases for which each transmitter has a separate BER curve, since the transmitters are independent from each other. It should be noted that the BER curve of the close transmitter is not affected by the first optimization problem. As observed from this figure, the performance for the closer transmitter of the first case is significantly better than the further one, meaning that the performance of this system exhibits unequal error protection for the two transmitters. For the second case, it can be observed that there is an improvement of the average BER curve for the two communication links. However, there is still room for improvement as the goal is to reach equal error protection, thus bringing the average BER curve even closer to the individual ones.

In order to observe the conditions of the two transmitters more clearly, the channel coefficients and CDF curves for the two aformentioned scenarios are also presented in Fig. 3 and 4, respectively. It is observed that the error between the two CDF curves is significantly reduced for the optimized D_2 value. As the first optimization solution aims, the first channel taps of the two transmitters are matched, which is reflected on their CDF curves as well. The aforementioned improvement on the BER performance of the far transmitter is expected, since its first channel tap is increased and the following channel taps, which account for ISI, are slightly decreased.

For the third case, the initialization values of M_2 and D_2

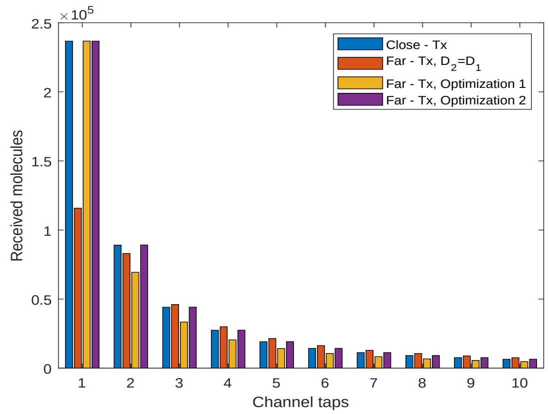

Fig. 3. Channel coefficients for not optimized and optimized cases.

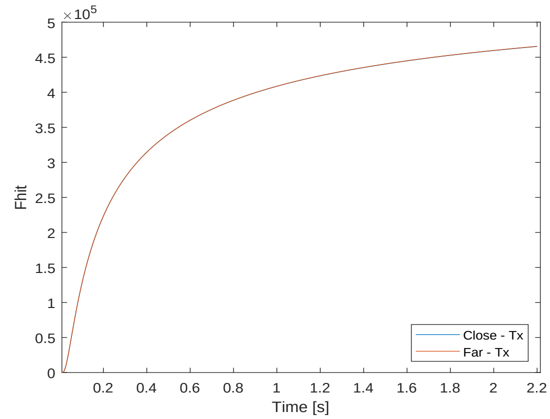

Fig. 5. CDF curves for optimized D_2 and M_2 values.

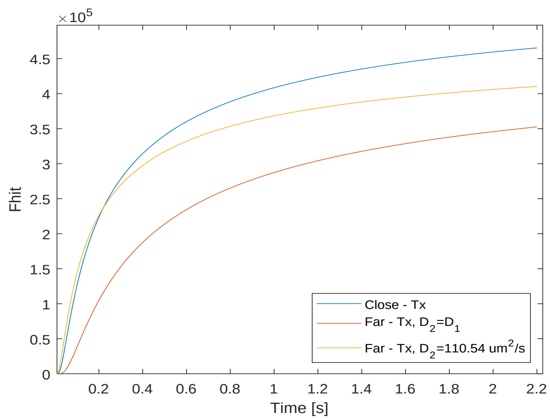

Fig. 4. CDF curves for not optimized and optimized D_2.

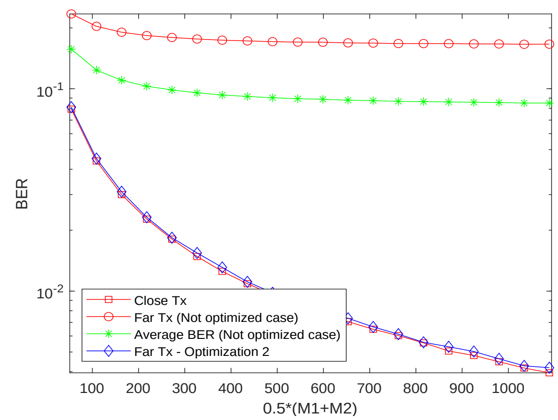

Fig. 6. BER performances of the two communication links for not optimized parameters and the optimized D_2 and M_2 case.

are selected to be 10^6 and $79.4\mu m^2/s$, respectively, with the latter being the benchmark value of diffusion coefficient in the literature, whereas all the other parameters are the same as before. After seven iterations, the values converged to $M_2 = 1.17738 \cdot 10^6$ and $D_2 = 80.9126\mu m^2/s$. For the newly obtained values, the CDFs and BER performance of the scheme are illustrated in Figs. 5 and 6, respectively, whereas the channel coefficients are shown in Fig. 3. It is observed that for these values, the CDF curves and all the channel coefficients for both transmitters are equal. This is clearly reflected in the BER performances as well, where the average BER curve is very close to the the curves of the individual molecular links. In other words, the optimized parameters enable the system to achieve equal error protection for both communication links.

V. Conclusion

In this work, asymmetric communication links of the uplink of a relay scheme have been investigated, for which error protection and communication fairness of these links have been improved by considering the proposed optimization solutions. Two solutions, which optimize the parameters of the further transmitter's communication link, have been presented. The first one is based on optimizing its diffusion coefficient only given the other system parameters, whereas the other takes into account both the diffusion coefficient and the number of emitted molecules. In order to compare the obtained results, channel coefficients, CDF, and BER curves have been provided. We note that our solutions can be similarly extended to the downlink of a relaying system, due to the channel symmetry. However, for simplicity purposes, only uplink transmission is considered in this work. Moreover, this idea could be further extended to a relay scheme with higher number of transmitters as well as it could be incorporated to the IM technique.

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