Abstract—In this paper, power allocation problem for cooperative THz MIMO-NOMA system is studied to maximize the minimum achievable rate of the available users. The cooperative system consists of a base station (Bs) that communicates directly with a nearby user, while a relay node (RN) is used to help the Bs to communicate with the far user. The RN operates in HD and FD modes using DF protocol. The Bs is equipped with multiple antennas, while both the relay and the user nodes are equipped with single antenna. The optimization problem for power allocation is solved analytically and validated numerically.

I. INTRODUCTION

Terahertz (THz) communications have been considered as a promising technique for future wireless networks because of ultrahigh bandwidth. The THz band can achieve up to \((10^4)\) higher magnitude than the current wireless communication band commonly used in mobile phones, which can provide huge communication bandwidth. In addition, multiple-input multiple output (MIMO) technology achieved the high data rate demand in 5G and beyond. On the other hand, non-orthogonal multiple access (NOMA) schemes allow multiple users to share the same resources (e.g. a time/frequency resource block) to increase the network capacity [1]. The authors in [2] proposed a fast convergence scheme for user clustering in NOMA-MIMO system in THz band, by using enhanced K-means machine learning algorithms. The author in [3] considered the MIMO systems and sub-TeraHertz (sub-THz) bands are being for the development of ultra-high data rate applications in beyond 5G. It is well known that the very large available bandwidths at THz frequencies come at the cost of severe propagation and power losses. To overcome this problem, ultra-massive MIMO technology is considered in [4] to overcome the distance problem at the terahertz (THz) band. Recently, cooperative NOMA schemes have attracted a lot of attention, and a variety of problems have been studied. Power allocation plays a very important role in traditional NOMA systems as well as cooperative NOMA systems. Hence, many researches focus on this issue, and various power allocation policies have been proposed with different optimization objectives for diverse NOMA systems. The authors in [5] propose a two-phase FD C-NOMA system where the Bs only transmits a new signal for the strong user in the second phase, such that the weak user can perform SIC to obtain its own signal. Furthermore, the work in [6] investigates the performance of cooperative networks based on non-orthogonal multiple (NOMA) with multiple FD decode-and-forward relay in order to maximize the minimum users’ achievable rate. The authors in [7] studied multiple relay selection strategy and comparison of different diversity combining techniques at the receiver in NOMA based wireless cooperative networks to enhance the symbol error rate (SER) performance. From the above discussion, the researchers do not focus on power allocation optimization of cooperative MIMO-NOMA in THz band which is the main focus of this paper.

II. SYSTEM MODEL

Consider a single cell cooperative THz MIMO-NOMA in system model as shown in Fig. 1 and 2, the main base station is equipped with multiple antennas (N). The Bs is linked to two
users; assuming that the first user (UE1) is not in the coverage area of the Bs, an intermediate relay node is used to help in linking the Bs with the far user UE1. The second user (UE2) directly connected to the Bs. Downlink model is assumed where UE1 and UE2 are the receivers and Bs operates as the transmitter. The relay node is assumed to use the DF protocol. Accordingly, the Bs uses NOMA technology to improve the capacity of the network. It is assumed that there is no direct ink between Bs and UE1 due to the high pathloss.

A. Terahertz Channel

Channel model of THz band is developed by using THz wave atmospheric transmission attenuation model and experiential water vapour continuum absorption. In THz band, because the pathloss of non line of site link (NLOS) is much larger than line of site link (LOS), the influence of NLOS can be neglected when LOS link exists [8]. The channel gain of the kth user can be formulated as

$$h_k = \sqrt{\frac{1}{PL(f, d)}} \alpha a(\theta)$$

where $PL(f, d)$ stands for the pathloss determined by THz frequency $f$ and distance $d$ between Bs and user. $\alpha$ is the antenna gains and $a(\theta)$ is the array steering vector which is given by: $a(\theta) = [1, ..., e^{\jmath \pi \sin(\theta)}, ..., e^{\jmath \pi \sin(\theta)(N-1)}]$. In particular, the path gain consists of spreading loss and molecular absorption which cannot be neglected in THz band. The spreading loss is caused by the expansion of electromagnetic wave as it propagates through various mediums. The molecular absorption attenuation is a result of the collisions initiated by atmospheric gas or water molecules. Further, the pathloss of frequency $f$ suffers when travelling a distance $d$ which can be expressed by:

$$PL(f, d) = \frac{4\pi fd}{c}e^{k(f)d}$$

where $c$ is the speed of light in free space, $k(f)$ is frequency-dependent medium absorption coefficient.

B. Achievable Rates for Cooperative MIMO-NOMA using THz with HD Relaying

1) HD Phase 1: The Bs utilizes power NOMA technology, so it combines the two signals $x_1$ and $x_2$ with power coefficients $a_{11}$ and $a_{12}$ respectively. The user UE1 is interested in signal $x_1$, while the user UE2 is interested in signal $x_2$. Thus the combined signals received by the RN can be written by:

$$y_{RN-1} = h_{SR}^{T}[\sqrt{a_{11}P_s}x_1 + \sqrt{a_{12}P_s}x_2] + n_{RN}$$

where $h_{SR}$ is a (1xN) channel response vector between Bs and RN, $a_{11}$ is the power coefficient allocated for the (N1) vector message $x_1$, while $a_{12}$ which is the power allocation coefficient for the (Nx1) vector message $x_2$. $P_s$ is the total transmitted power at the Bs and $n_{RN}$ is the additive noise AWGN at the RN. Similarly, the received signal at UE2 is given by:

$$y_{UE2-1} = h_{S2}^{T}[\sqrt{a_{11}P_s}x_1 + \sqrt{a_{12}P_s}x_2] + n_{UE2}$$

where, $h_{S2}$ is the (1xN) THz channel response vector between Bs and UE2. Using (3) the SINR at the RN can be expressed by:

$$SINR_{RN-1} = \frac{\|h_{SR}\|^2a_{11}P_s}{\|h_{SR}\|^2a_{12}P_s + \sigma_{RN}^2}$$

where $\sigma_{RN}^2$ is the noise power at the RN. It is noted that the signal $x_2$ is considered as an interference since the RN is only interested in $x_1$ to forward it to UE1. In the case of obtaining the SINR at UE2, successive interference cancellation (SIC) is applied which guarantees that the second user (UE2) first decodes $x_1$ and then subtracts it from the overall received message and after that decodes the desired message $x_2$. Consequently, in this case UE2 can remove the interference $x_1$ and the SINR for UE2, using (4), is given by:

$$SINR_{UE2-1} = \frac{\|h_{S2}\|^2a_{12}P_s}{\sigma_{UE2}^2}$$

where $\sigma_{UE2}^2$ is the noise power at UE2. Using (5), the achievable rate at the RN can be expressed as:

$$R_{RN-1} = 1/2 log_2 \left(1 + \frac{\|h_{SR}\|^2a_{11}P_s}{\|h_{SR}\|^2a_{12}P_s + \sigma_{RN}^2} \right)$$

Similarly the achievable rate at UE2 is given as:

$$R_{UE2-1} = 1/2 log_2 \left(1 + \frac{\|h_{S2}\|^2a_{12}P_s}{\sigma_{UE2}^2} \right)$$

The pre log factor $1/2$ results from the half duplex mode.

2) HD Phase 2: After receiving the messages from Bs, RN sends the message $x_1$ to UE1 after decoding and extracting the desired signal from the combined signal, using DF protocol. In phase 2, the Bs sends the message again to UE2, the sent message contains $x_2$ only, which can help in better reception for the message at UE2. The received message at UE1 from RN can be expressed as:

$$y_{UE1-2} = h_{R1}^{T}\sqrt{a_{21}P_s}x_1 + n_{UE1}$$

where $a_{21}$ is the power allocation factor for the message $x_1$ sent from RN to UE1 in phase 2, $x_1$ is the received message by UE1 and finally $n_{UE1}$ is the AWGN noise at UE1. Similarly, the received signal at UE2 from the Bs can be expressed by:

$$y_{UE2-2} = h_{R2}^{T}[\sqrt{a_{21}P_s}x_1] + h_{S2}^{T}[\sqrt{a_{22}P_s}x_2] + n_{UE2}$$

where $a_{22}$ is the power allocation coefficient for the message $x_2$.

From (9), the SINR for UE1 can be written as

$$SINR_{UE1-2} = \frac{|h_{R1}|^2a_{21}P_s}{\sigma_{UE1}^2}$$
Consequently, using (10) and after using SIC to remove the interference message $x_1$, the SINR at UE2 can be written as:

$$\text{SINR}_{UE2} = \frac{\|h_{s2}\|^2 a_{22} P_s}{\sigma_{UE2}^2}$$  \hspace{1cm} (12)

The achievable rates for UE2 and UE1 respectively at phase 2 can be expressed as:

$$R_{UE1-2} = \frac{1}{2} \log_2 \left( 1 + \frac{\|h_{r1}\|^2 a_{21} P_s}{\sigma_{UE1}^2} \right)$$  \hspace{1cm} (13)

$$R_{UE2-2} = \frac{1}{2} \log_2 \left( 1 + \frac{\|h_{s2}\|^2 a_{22} P_s}{\sigma_{UE2}^2} \right)$$  \hspace{1cm} (14)

The total achievable rates at the end of the two phases at UE1 and UE2, respectively can be given by:

$$R_{UE1} = \frac{1}{2} \log_2 \left( 1 + \min \left( \frac{\|h_{r1}\|^2 a_{21} P_s}{\sigma_{UE1}^2}, \frac{\|h_{s1}\|^2 a_{21} P_s}{\sigma_{UE1}^2} + \frac{\|h_{sr}\|^2 a_{21} P_s}{\sigma_{UE1}^2} + \frac{\|h_{sr}\|^2 a_{21} P_s}{\sigma_{UE1}^2} \right) \right)$$  \hspace{1cm} (15)

$$R_{UE2} = \frac{1}{2} \log_2 \left( 1 + \frac{\|h_{s2}\|^2 a_{22} P_s}{\sigma_{UE2}^2} \right) + \frac{1}{2} \log_2 \left( 1 + \frac{\|h_{s2}\|^2 a_{22} P_s}{\sigma_{UE2}^2} \right)$$  \hspace{1cm} (16)

Our objective is to maximize the achievable rates in equations (15) and (16) which depends mainly on optimizing the power allocation coefficients $\{a_{11}, a_{12}, a_{21}, a_{22} \}$ under certain constraints.

C. Optimal Power Allocation for Cooperative MIMO-NOMA in HD Relaying

1) HD Relaying Power Allocation Optimization Phase 1: An optimization problem is formulated to maximize the minimum achievable rates as following:

$$\max_{\mathbf{P}_h} \min \{ R_{RN-1}, R_{UE2-1} \}$$

s.t. \hspace{1cm} $C0 : a_{11} + a_{12} \leq 1$

$$C1 : P_{h1} \geq 0$$  \hspace{1cm} (17)

where $\mathbf{P}_{h1}$ is a vector containing all the power allocation coefficients $\mathbf{P}_{h1} = [a_{11}, a_{12}]$, $R_{RN-1}$ and $R_{UE2-1}$ are given respectively by (7) and (8). The optimization problem is neither convex nor concave, in order to prove the quasi-concavity of the optimization problem, the constraints must be convex along with the superlevel set of the objective function [11]. It is observed that the above optimization problem constraints C0 and C1 are convex due to their linearity. To prove the quasi-concavity of our optimization problem, the superlevel sets $S_{\beta h} = \{ \mathbf{P}_{h1} | R_{RN-1}, R_{UE2-1} > \beta h \}$ must be convex, where $\beta h$ is an optimization threshold factor. After mathematical calculations, the constraints of the superlevel sets can be written as

$$\frac{\|h_{sr}\|^2 a_{11} P_s}{\|h_{sr}\|^2 a_{12} P_s + \sigma_{RN}^2} \geq \beta h$$  \hspace{1cm} (18)

$$\frac{\|h_{s2}\|^2 a_{12} P_s}{\sigma_{UE2}^2} \geq \beta h$$  \hspace{1cm} (19)

which can be reformulated to:

$$\|h_{sr}\|^2 a_{11} P_s \geq \beta h (\|h_{sr}\|^2 a_{12} P_s + \sigma_{RN}^2)$$  \hspace{1cm} (20)

Hence, all the superlevel sets are convex because they can be expressed as the intersection of two convex sets. Therefore, the objective function is quasi-concave. The optimization problem can be rewritten as:

$$\text{find : } \mathbf{P}_{h1}$$

s.t. \hspace{1cm} $C0 : a_{11} + a_{12} \leq 1$

$$C1 : P_{h1} \geq 0$$

$$C2 : R_{RN-1} \geq \beta h$$

$$C3 : R_{UE2-1} \geq \beta h$$  \hspace{1cm} (22)

By solving both (20) and (21), $a_{11}$ and $a_{12}$ are given by:

$$a_{11} = \beta h \left( \frac{\|h_{sr}\|^2 a_{12} P_s + \sigma_{RN}^2}{\|h_{sr}\|^2 P_s} \right)$$  \hspace{1cm} (23)

$$a_{12} = \frac{\beta h \sigma_{UE2}^2}{\|h_{s2}\|^2 P_s}$$  \hspace{1cm} (24)

Therefore (23) and (24) are the optimized power allocation factors in terms of the optimization variable $\beta h$. The optimization variable $\beta h$ can be optimized by assuming that $a_{11} + a_{12} = 1$ which is the maximum optimization constraint of C0 in (22). By substituting (23) and (24) in C0 constraint, we have,

$$\frac{\beta h \sigma_{UE2}^2 P_s + \beta h (\|h_{sr}\|^2 a_{12} P_s + \sigma_{RN}^2)}{\|h_{sr}\|^2 P_s} \geq \beta h$$

By further simplifications:

$$\beta h \sigma_{UE2}^2 P_s + \beta h (\|h_{sr}\|^2 a_{12} P_s + \sigma_{RN}^2) \geq \beta h \sigma_{RN}^2 \|h_{s2}\|^2 P_s$$

By substituting (24) in (26)

$$\beta h \sigma_{UE2}^2 P_s + \beta h (\|h_{sr}\|^2 a_{12} P_s + \sigma_{RN}^2) \geq \beta h \sigma_{RN}^2 \|h_{s2}\|^2 P_s$$

By re-arranging (27) into a quadratic formula,

$$\beta h \left( \sigma_{UE2}^2 \|h_{sr}\|^2 P_s + \beta h \sigma_{UE2}^2 \|h_{sr}\|^2 a_{12} P_s + \sigma_{RN}^2 \|h_{s2}\|^2 P_s \right) + (-\|h_{sr}\|^2 \|h_{s2}\|^2 P_s^2) = 0$$  \hspace{1cm} (28)

Therefore, the optimum solution for $\beta h$ can be solved using quadratic formula,

$$\beta h = \frac{A + \sqrt{B^2 - 4AC}}{2A}$$  \hspace{1cm} (29)

where $A = \sigma_{UE2}^2 \|h_{sr}\|^2 P_s$, $B = \sigma_{UE2}^2 \|h_{sr}\|^2 a_{12} P_s + \sigma_{RN}^2 \|h_{s2}\|^2 P_s$, and $C = -\|h_{sr}\|^2 \|h_{s2}\|^2 P_s^2$.

After reaching the optimum solution for $\beta h$ in (29), we substitute in (23) and (24) in order to get the optimum power allocation coefficients that will optimize the minimum achievable rates.
2) HD Relaying Power Allocation Optimization Phase 2:
Following the same steps similar to phase 1, the optimization problem is quasi-concave as mentioned in the last section. The optimized power allocation factors can be written as:

\[ a_{21} = \frac{\beta_2 a_{1}^2}{|h_{R1}|^2 P_s} \]  \hspace{1cm} \text{(30)}

\[ a_{22} = \frac{\beta_2 a_{2}^2}{|h_{S2}|^2 P_s} \]  \hspace{1cm} \text{(31)}

The optimization factor \( \beta_2 \) can be written as:

\[ \beta_2 = \frac{|h_{R1}|^2 |h_{S2}|^2 P_s^2}{\sigma_{U1}^2 P_s + \sigma_{U2}^2 |h_{R1}| P_s} \]  \hspace{1cm} \text{(32)}

After reaching the optimum \( \beta_2 \) in (32), we substitute with (32) in (31) and (30) in order to get the optimum power allocation coefficients.

D. Achievable Rates for Cooperative NOMA with FD Relaying

The received signal at RN from the Bs can be written as:

\[ y_{RN} = h_{SR}^T [\sqrt{a_1 P_s} x_1 + \sqrt{a_2 P_s} x_2] + h_{RR} P_s x_1 + n_{RN} \]  \hspace{1cm} \text{(33)}

where \( P_s \) is the power of the RN which can be expressed as \( a_3 P_s \), so the total power is split between the Bs and the RN. Moreover, \( a_1 \) and \( a_2 \) are the power allocation coefficients of \( x_1 \) and \( x_2 \) respectively, \( h_{SR}^T \) is the (1xN) THz channel vectors received at RN and \( h_{RR} \) is the self interference 1x1 channel. The received signal at UE1 can be written by:

\[ y_{UE1} = h_{R1} \sqrt{a_3 P_s} x_1 + n_{UE1} \]  \hspace{1cm} \text{(34)}

The received signal at UE2 can be written by:

\[ y_{UE2} = h_{S2}^T [\sqrt{a_1 P_s} x_1 + \sqrt{a_2 P_s} x_2] + h_{S2} P_s x_1 + n_{UE2} \]  \hspace{1cm} \text{(35)}

where \( h_{S2}^T \) is the (1xN) THz channel vector received at UE2. The SINR at RN, UE1 and UE2 using (37),(38) and (39) can be written as:

\[ \text{SINR}_{RN} = \frac{|h_{SR}|^2 a_1 P_s}{(|h_{SR}|^2 a_2 P_s) + (|h_{RR}|^2 a_3 P_s) + \sigma_{RN}^2} \]  \hspace{1cm} \text{(36)}

\[ \text{SINR}_{UE1} = \frac{|h_{R1}|^2 a_3 P_s}{\sigma_{U1}^2} \]  \hspace{1cm} \text{(37)}

\[ \text{SINR}_{UE2} = \frac{|h_{S2}|^2 a_2 P_s}{\sigma_{U2}^2} \]  \hspace{1cm} \text{(38)}

using (36),(37) and (38), the achievable rate at the RN, UE1 and UE2 can be expressed by:

\[ R_{RN} = \log_2 \left( 1 + \frac{|h_{SR}|^2 a_1 P_s}{(|h_{SR}|^2 a_2 P_s) + (|h_{RR}|^2 a_3 P_s) + \sigma_{RN}^2} \right) \]  \hspace{1cm} \text{(39)}

\[ R_{UE1} = \log_2 \left( 1 + \frac{|h_{R1}|^2 a_3 P_s}{\sigma_{U1}^2} \right) \]  \hspace{1cm} \text{(40)}

\[ R_{UE2} = \log_2 \left( 1 + \frac{|h_{S2}|^2 a_2 P_s}{\sigma_{U2}^2} \right) \]  \hspace{1cm} \text{(41)}

The achievable rate at UE1 considering DF protocol used at the RN can be given by:

\[ R_{UE1} = \log_2 \left( 1 + \min \left( \frac{|h_{R1}|^2 a_1 P_s}{\sigma_{U1}^2}, \frac{|h_{SR}|^2 a_2 P_s}{\sigma_{U2}^2} \right) \right) \]  \hspace{1cm} \text{(42)}

In the subsequent section, power optimization algorithm is derived to maximize the derive achievable rates for the FD mode.

E. Optimal Power Allocation for Cooperative-NOMA with FD Relaying

The optimization problem in FD can be written as:

\[ \max_{P_f} \min(R_1, R_2) \]  \hspace{1cm} \text{s.t.} \hspace{1cm} \text{C0 : } a_1 + a_2 + a_3 \leq 1 \hspace{1cm} \text{C1 : } P_f \geq 0 \]  \hspace{1cm} \text{(43)}

Where \( P_f \) is a vector containing all the power allocation coefficients \( P_f = [a_1, a_2, a_3] \). The optimization problem is solved similarly as to the HD mode. The optimized power allocation coefficients in the FD mode can be written as:

\[ a_1 = \frac{\beta_f (|h_{SR}|^2 a_2 P_s + |h_{RR}|^2 a_3 P_s + \sigma_{RN}^2)}{|h_{SR}|^2 P_s} \]  \hspace{1cm} \text{(44)}

\[ a_2 = \frac{\beta_f |h_{S2}|^2 P_s}{|h_{SR}|^2 P_s} \]  \hspace{1cm} \text{(45)}

\[ a_3 = \frac{\beta_f |h_{U1}|^2 P_s}{|h_{R1}|^2 P_s} \]  \hspace{1cm} \text{(46)}

The solution for the optimum \( \beta_f \) can be written as:

\[ \beta_f = \frac{A + \sqrt{B^2 - 4AC}}{2A} \]  \hspace{1cm} \text{(47)}

where \( A = (|h_{SR}|^2 |h_{SR}|^2 |h_{RR}|^2 + P_s^2 |h_{SR}|^2 + P_s^2 |h_{SR}|^2 + P_s^2 |h_{SR}|^2 + P_s^2 |h_{SR}|^2 + P_s^2 |h_{SR}|^2)^2 \), \( B = (|h_{SR}|^2 |h_{SR}|^2 |h_{SR}|^2 + P_s^2 |h_{SR}|^2 + P_s^2 |h_{SR}|^2 + P_s^2 |h_{SR}|^2 + P_s^2 |h_{SR}|^2 + P_s^2 |h_{SR}|^2)^2 \) and \( C = -P_s^2 |h_{R1}|^2 |h_{SR}|^2 + P_s^2 |h_{SR}|^2 + P_s^2 |h_{SR}|^2 \).

III. NUMERICAL SIMULATIONS

In this section, numerical analysis and simulations are carried out to validate the optimized total achievable rates. The Bandwidth used is 300 MHz, the absorption coefficient \( k(f) \) can be found in [9], and the normalized distance used is 0.7 between Bs and RN, 0.3 between Bs and UE2 and 0.3 between RN and UE1. The frequency used in THz communication transmission is 0.3 THz and finally the antenna gain is assumed to be 25 dB.

Fig.3 shows the sum rate of the system versus the power \( P_s \) for different number of antennas in HD mode. Fixed power allocation is included in the figure for comparison purposes [10]. The figure shows that the optimal power allocation outperforms the fixed one in terms of the sum rate. However when the number of antennas increases to 50, the fixed power allocation is found to have the same performance as the
optimum power allocation. This behaviour occurs because as
the number of antennas increases, the power transmitted from
Bs to UE2 increases, however, the power of the RN remains
constant. Thus, the system provides a higher power allocation
factor $a_{22}$ for the RN and decreases the transmitting power
allocation $a_{21}$ at the Bs to UE2. Similarly Fig. 4 is obtained for
the FD relaying mode, with a $0dB$ self interference power. The
optimal power allocation outperforms the fixed model up to 50
antennas, however, when the number of antennas increases to
50, again the fixed model has almost the same performance as
the optimal model for the same reason mentioned in HD case.
The fixed power allocation variables where $a_1 = \frac{19}{30}, a_2 = \frac{1}{3}$
and $a_3 = \frac{10}{30}$. Fig.5 and 6 show the sum rate versus distance
between Bs and RN and between Bs and UE2 respectively.
while Fig.7 and 8 show the sum rate for FD mode versus
the same distance with $0dB$ self interference power. Results
show that by increasing both distances, the overall sum rates
decrease and this is due to the high propagation loss of THz
channel. However, it is shown that the distance between Bs and
RN has a much higher effect on the overall sum rate than the
increasing distance between Bs and UE2. The reason for this
is that the relay node in phase 2 has higher power allocation
factor $a_{21}$ than the Bs $a_{22}$ in HD and $a_3$ in FD to compensate
the high propagation loss between RN and UE1.
Results also show that the sever attenuation of the sum rate with distance is resolved by increasing the number of antennas. As a comparison between HD and FD, it is shown that the rate of decaying of the sum rate in HD is higher than FD because in case of HD, the power is split between two links in one phase, however in FD the power is split between 3 links. Thus, the effect of decreasing one power allocation variable in HD is higher than the FD.

IV. CONCLUSION

A cooperative MIMO-NOMA THz system model with HD/FD relaying has been investigated. The optimal power allocation of the NOMA coefficients has been derived to maximize the achievable data rates for both HD and FD modes. Numerical results showed a comparison between the optimal power allocation scheme and a fixed scheme. The optimal scheme outperformed the fixed scheme in terms of the sum rate in HD/FD mode. However, as the number of antennas increases, the fixed model and the optimal model had the same performance. The effect of the distancing between the available nodes was studied along with the number of antennas. Numerical result showed that in HD mode, the distance between Bs and RN has much higher effect than the distance between Bs and UE2.

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