Energy-Efficient Cooperative Spectrum Sensing based on Stochastic Programming in Dynamic Cognitive Radio Sensor Networks

HECTOR KASCHEL1, (Senior Member, IEEE), KAREL TOLEDO2, JORGE TORRES GÓMEZ3, (MEMBER, IEEE), AND M. JULIA FERNÁNDEZ-GETINO GARCÍA4, (Member, IEEE).

1Department of Electrical Engineering, University of Santiago de Chile, 9160000, Santiago, Chile (e-mail: hector.kaschel@usach.cl)
2Department of Electronics, Federico Santa María Technical University, 2390123, Valparaiso, Chile (e-mail: karel.tdlg@gmail.com)
3Department of Telecommunication Systems, Technical University of Berlin, 10587 Berlin, Germany (e-mail: jtorres151184@gmail.com)
4Department of Signal Theory and Communications, Carlos III University of Madrid, 28911, Leganés, Madrid, Spain (e-mail: mjulia@tsc.uc3m.es)

Corresponding author: Karel Toledo (e-mail: karel.tdlg@gmail.com).

This work was supported in part by the DICYT Project, Code 061813KC, Direction of Research, Development and Innovation, University of Santiago de Chile, USACH, the CONICYT-PFCHA/Doctorado Nacional/2016-21160292 and the Spanish National Project TERESA-ADA (TEC2017-90093-C3-2-R) (MINECO/AEI/FEDER, UE).

ABSTRACT Nowadays, Cognitive Radio Sensor Networks (CRSN) arise as an emergent technology to deal with the spectrum scarcity issue and the focus is on devising novel energy-efficient solutions. In static CRSN, where nodes have spatial fixed positions, several reported solutions are implemented via sensor selection strategies to reduce consumed energy during cooperative spectrum sensing. However, energy-efficient solutions for dynamic CRSN, where nodes are able to change their spatial positions due to their movement, are nearly reported despite today’s growing applications of mobile networks. This paper investigates a novel framework to optimally predict energy consumption in cooperative spectrum sensing tasks, considering node mobility patterns suitable to model dynamic CRSN. A solution based on the Kataoka criterion is presented, that allows to minimize the consumed energy. It accurately estimates -with a given probability- the spent energy on the network, then to derive an optimal energy-efficient solution. An algorithm of reduced-complexity is also implemented to determine the total number of active nodes improving the exhaustive search method. Proper performance of the proposed strategy is illustrated by extensive simulation results for pico-cells and femto-cells in dynamic scenarios.

INDEX TERMS Dynamic CRSN, energy efficiency, spectrum sensing, stochastic programming.

I. INTRODUCTION

Cognitive Radio (CR) constitutes a growing technology to overcome the increased spectrum occupancy for telecommunication services [1], [2]. This paradigm has been introduced in Wireless Sensor Networks to deal with frequency bands scarcity in industrial, scientific, and medical bands. As a result, a new network paradigm, called Cognitive Radio Sensor Networks (CRSN), has increased interest to implement promising solutions for Internet of Things (IoT) applications provided its capability to manage the spectrum resources wisely [3]–[5].

A CRSN implements the dynamic access to available network resources through a self-organized approach. The availability of network resources, regarding bandwidth, is determined through Spectrum Sensing (SS) techniques to detect spectrum holes and avoid interference. It is comprised of small devices to support CR capabilities, namely Secondary Users (SUs); devices with legacy rights on spectrum usage called Primary Users (PUs); and a fusion center (FC), who merges the received information from SUs to have a final decision about spectrum bands availability through Cooperative Spectrum Sensing (CSS). Here we consider applications for sensor networks where the SUs are running on general-purpose sensor nodes.

Reducing energy consumption is one of the greatest challenges in CRSN provided the inclusion of spectrum sensing
techniques in addition to the usual sensor node operations regarding transmission and reception of information. Neither transmit nor receive information is currently enough, but to find spectrum holes -employing spectrum sensing techniques- to increase transmission opportunities is also of major importance. However, the implementation of spectrum sensing capabilities will demand to increase the energy consumption of nodes, which in turn will reduce the network lifetime. To overcome these inconveniences, reported solutions address the problem from two viewpoints mainly: energy harvesting [6], [7] and energy conservation [8], [9]. Both approaches aim to ensure the successful performance of network nodes, as long as possible, to operate on unattended mode bases.

Energy harvesting solutions assume that the nodes obtain energy autonomously and then propose optimal schedules to perform sensing operations based on the dynamics of the gathered energy values [10]. In a different approach, energy conservation techniques deal with extending the current battery level in sensor nodes with energy-efficient cognitive radio capabilities. These solutions focused on the following directions: maximizing the ratio of the throughput to the energy consumption [8], [11], finding the optimal power allocation strategy between the network-nodes [9], implementing energy efficient spectrum sensing policies at each sensor node [12], and the devising of sensor selection strategies [13]–[21], when CSS is implemented and energy constraints are imposed.

In specifics, sensor selection strategies will provide the preferred sensor nodes to participate in CSS. Remaining nodes will be on sleep mode to reduce energy consumption and extend the network lifetime. These solutions are based on computing the minimum number of awake nodes to run CSS and simultaneously satisfying a given detection performance.

Essentially, this is done by stating an optimal problem formulation to reduce the total consumed energy while guaranteeing detection performance and later solved by heuristic algorithms to devise a short-term solution of reduced time-complexity. The energy-consumption variable accounts for the channel sensing operations, the running of the decision rule, the signal processing to modulate and demodulate, as well as the energy used to report the resulting decision about the spectrum availability. Performance metrics are given by the detection and false alarm probabilities.

Considering the complexity to find the optimal solution by the exhaustive search algorithm, some heuristics have been reported to address the problem in practice. Departing from the equivalent Lagrangian formulation, a strategy for the sensor selection to participate in CSS is conceived by analyzing the contribution of each node to the total energy consumption utilizing weighted coefficients [13]. Those nodes with the higher coefficient will consequently employ less energy when computing and reporting the spectrum sensing results.

Besides, sensor selection strategies are also reported to account for a balanced energy-level criteria regarding the node’s battery level, which in turn will imply a more appropriate sensor selection strategy [14]–[18]. The remaining energy per node may account for the priority to participate in SS, those with the higher levels are preferred to participate in SS [14], [17]. More elaborated solutions are also devised by considering not-complete information about the network status [21], or after clustering nodes according to their detection capabilities. In order to extend the network lifetime, mechanisms for equal energy consumption are achieved by engaging the participation of sensors with a reduced probability of detection to participate in SS. This will avoid a rapid battery depleting of such sensors with the higher performance [15], [16], [18].

The SS capabilities regarding each sensor node can be also improved to reduce the demands of additional active nodes performing SS operations. This can be achieved through the use of multiples antennas to improve the detection performance in the low signal-to-noise ratio (SNR) regime [16], [19], or by computing the optimal threshold detection [17] to operate with enhanced capabilities.

However, mobility of nodes -an essential issue in mobile networks for IoT applications [22], [23] and Internet of Mobile Things environments [24], [25]- modifies the network topology dynamically, which in turn will limit the application of reported static solutions regarding sensor selection for CSS. Static approaches posed in [13]–[21] are insufficient to extend the network lifetime in dynamic CRSN due to the inherent assumption of constant distances from each sensor node to the FC and the PU nodes. In this direction, these solutions are repeatedly applied in time-slots to obtain an optimal solution in an attempt to discretize the time evolution regarding the network dynamics. These concerns encourage the further extent of energy-consumption based strategies to properly consider the randomness of dynamic CRSNs.

Since distances will not be fixed but inherently random in dynamic networks, sensor selection strategies for mobile nodes can be addressed through stochastic programming techniques by means of two approaches: “wait-and-see” [26] and ”here-and-know” [27]. These approaches formulate solutions to optimization problems involving random variables (in our case the random position of nodes).

“Wait-and-see” approach computes the total number of awake nodes per time-slot, i.e, the problem is solved by applying the tools of static solutions as discussed above. This method is not particularly suited for dynamic scenarios, due to the repeated computation of the optimal solution whenever the sensor nodes change positions. This would imply repeatedly applying the optimization algorithm (at the beginning of each time-slot) to select the preferred nodes to participate in CSS with the corresponding waste of energy [28].

On the other hand, “here-and-now” allows to devising solutions without the specifics of nodes location relying on the statistical description of movement instead. Although sub-optimal, the solution reported in [29] exploits the statistical metrics to compute only once -when the network starts the running operation- the total number of nodes to participate in CSS exhibiting less power consumption than
the optimal solution “wait-and-see”. This becomes a more suitable approach considering that the solution will be not recomputed each time nodes change their positions. However, this method implements a pessimistic estimate of the energy consumption values because its general formulation relies on Chebyshev’s inequality, which in turn will overestimate the network resources. A more accurate estimation of the energy consumption would imply a better resource estimation for the network operation.

In this regard, current work addresses a sub-optimal solution based on “here-and-now” approach to accurately estimate network energy consumption values in dynamic CRSN. We develop a novel stochastic model to study energy performance by considering the impact of different mobility patterns of participating sensor nodes. Main contributions of this paper are listed as follows:

- We model a dynamic CRSN taking into account the random movement behavior of nodes. Then, we derive a novel strategy to forecast the energy consumption on CSS based on a stochastic optimization approach. The solution to this problem stems from the application of “here-and-now” stochastic approach based on the Kataoka criterion, which derives a more accurate solution through the cumulative distribution function of the distances between each sensor node and the FC.
- A new iterative algorithm is developed to select the optimal number of awake sensor nodes for CSS while remaining nodes stay in sleep mode to save energy on batteries. This algorithm avoids the repeated application of the static solution whenever nodes change positions. Besides, the resulting computational complexity is improved concerning the exhaustive search algorithm.
- We minimize the consumed energy in CSS avoiding to update nodes’ position and also, resource allocation of network devices can be improved provided the energy is accurately estimated for diverse scenarios. We consider the cases of different network sizes, positions of the PU, and three mobility models: Random Walk, Random Waypoint, and Gauss-Markov. Through Monte Carlo simulations, we demonstrate that global detection constraints are fulfilled on CRSN of reduced dimensions when the communication link is corrupted by Additive White Gaussian Noise.

This paper is structured as follows. System model and detection theory are presented in Section II. Problem formulation and energy metric related to CSS are discussed in Section III. Proposed solution based on stochastic optimization approach and the corresponding iterative algorithm are introduced in Section IV and V, respectively. A case of study for three mobility models is presented in Section VI. Section VII provides illustrative examples for a variety of simulation scenarios followed by the concluding remarks in Section VIII.

II. SYSTEM MODEL

We consider a CRSN composed of \( N \) sensor nodes that perform random movements on a given square field of side \( s \), as shown in Fig. 1. At first, nodes are uniformly distributed over the square field, a Fusion Center (FC) is located at the center position, and the PU is located outside of the field. Each sensor node moves randomly to a different position on a bounded area following a given mobility model (to be discussed in Section VI).

Sensor nodes support cognitive radio capabilities to handle the spectrum scarcity problem. Besides, sensor nodes send local spectrum sensing information to a given FC, who merges collected data to take a final decision about spectrum bands’ availability. Channel status needs to be properly determined to prevent interference with the PU signal. For each sensor node, the main parameters for CSS are given by the spectrum sensing duration \( \delta \) and the sampling frequency \( f_s \), which in turn will specify the total number of processed samples by \( \delta f_s \).

Typically, the energy detector is the sensing technique implemented for CSS, due to its reduced complexity and correspondingly reduced energy consumption [30]. This is the method implemented on each node to detect available spectrum bands. Statistical decision is made by following two hypothesis: \( H_1 \) and \( H_0 \). The first one, \( H_1 : y_j[n] = h_j[n] x_j[n] + u_j[n] \) represents busy channel due to the presence of the PU and the second one, \( H_0 : y_j[n] = u_j[n] \) represents idle channel due to the absence of the PU. The parameter \( n = \{1, 2, ..., \delta f_s\} \) is the time index, \( h_j[n] \) is the channel impulse response between each sensor node and the PU, \( x_j[n] \) is the signal transmitted from the PU, and \( u_j[n] \) is an i.i.d. Gaussian noise with zero mean and variance \( \sigma_u^2 \).

Based on energy detection principles, the decision rule can be stated as: \( H_1 \) if \( \mathcal{E}_j \geq \epsilon \) or \( H_0 \) if \( \mathcal{E}_j < \epsilon \). Parameter \( \epsilon \) represents the detection threshold and \( \mathcal{E}_j \) is the energy of the received signal at the \( j \)-th sensor defined by \( \mathcal{E}_j = \frac{1}{\delta f_s} \sum_{n=1}^{\delta f_s} |y_j[n]|^2 \). According to the Central Limit
Theorem, the distribution of $\mathcal{E}_j$ tends toward a Gaussian distribution when the number of samples becomes large. Consequently, false alarm and detection probabilities for a given $j$-th sensor are defined as follows [30]:

$$P_{f_j} = P(\mathcal{E}_j > \epsilon|\mathcal{H}_0) = Q\left(\frac{\epsilon}{\sigma^2_{u_j}} - 1\right)\sqrt{f_s},$$ (1)

$$P_{d_j} = P(\mathcal{E}_j > \epsilon|\mathcal{H}_1) = Q\left(\frac{\epsilon}{\sigma^2_{u_j}} - \gamma_j - 1\right)\sqrt{\frac{f_s}{2\gamma_j + 1}},$$ (2)

where $\gamma_j$ is the SNR between $j$-th sensor node and PU, and $Q$ is the complementary distribution function of the standard Gaussian distribution given by $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} dt.$

Each node sends one bit regarding the resulting sensing operations to the FC node. Then, the FC node implements a combined decision through AND, OR or Majority rules.

Due to the simplicity of OR rule [31], we shall adopt it to merge the information in the FC. By this rule a given frequency band is considered to be occupied if at least one sensor node claims the presence of the given PU, otherwise the frequency band is considered to be free for transmissions. By this rule, the global probability of false alarm $P_F$ and detection $P_D$ are given as follows [30]:

$$P_F = 1 - \prod_{j=1}^{N}(1 - \rho_j P_{f_j}),$$ (3)

$$P_D = 1 - \prod_{j=1}^{N}(1 - \rho_j P_{d_j}),$$ (4)

considering $\rho_j \in {0, 1}$, i.e. $\rho_j = 1$ when $j$-th sensor node participates in spectrum sensing, otherwise $\rho_j = 0$.

III. STOCHASTIC PROBLEM FORMULATION

Current work is focused on reducing the energy consumption on CSS and guaranteeing detection performance, simultaneously. To address this aim, a novel energy consumption model is proposed, where energy is divided in three main quantities: $E_{s_j}$ describes the total amount of consumed energy during sensing operations by each sensor in addition to the energy required to perform local decisions, $E_{t_j}$ indicates the consumed energy by each sensor node to report sensing operation results, and $E_{p_j}$ represents the consumed energy to report the position of each node. Therefore, the total energy consumption, denoted by $E_T$, will be given by:

$$E_T = \sum_{j=1}^{N} \rho_j(E_{s_j} + E_{t\text{-elec}} + e_{\text{amp}}d_j^2 + E_{p_j}),$$ (5)

where $E_{t\text{-elec}}$ stands for the energy dissipated to run the radio electronics, $e_{\text{amp}}$ is the required power amplification and $d_j$ is the distance between the $j$-th sensor node and FC. The parameter $\rho_j \in {0, 1}$ has been included in (5) to indicate which nodes will be awake or asleep.

Provided that each sensor node follows a random movement pattern (to be discussed in Section VI), the formulation in (5) will be stochastic-based. The mobility of sensor nodes will introduce random variability on the node distance to FC on each time-slot, given by term $d_j$ in (5), and then energy consumption values will behave randomly as well. Furthermore, here we will consider the typical case where the probability density function regarding the random movement of nodes is time-independent. This is the case of Random walk, Random Waypoint, and Gaussian-Markov (to be discussed in Section VI). In this regard, the variable $E_T$ will be a random variable as well.

To optimally reduce random energy consumption values we may state a stochastic formulation problem to properly select those sensor nodes that will be awake or asleep. At the same time, the network must satisfy given spectrum sensing performance metrics regarding false alarm and detection probabilities. By these considerations, the problem formulation is defined as follows:

$$\min_{\rho_j} E_T(\rho_j, d_j)$$ (6)

s.t. $P_F \leq \alpha, \quad P_D \geq \beta, \quad \rho_j \in {0, 1},$

where decision variable $\rho_j$ is a binary parameter, distance $d_j$ between each sensor node and FC is considered to be a random variable, which in turn implies that objective function given by $E_T$ is also a random variable. The constraints, regarding false alarm and detection global probabilities, must satisfy the threshold parameters $\alpha \in [0, 1]$ and $\beta \in [0, 1]$, respectively, to account for the proper CSS performance. Feasible solution to this problem is the vector $\rho = [\rho_j]$ for which the random variable $E_T(\rho_j, d_j)$ is minimized while simultaneously guaranteeing constraints on detection performance.

For the ease of mathematical tractability, here we assume that the values of $P_d$ and $P_f$ in (6) will be constant and independent of the node’s location. This assumption will be valid in fields of reduced length, which is typical for pico-cells and femto-cells in mobile networks [32], [33]. Otherwise, the varying position of nodes will randomly evaluate the values of $P_d$ and $P_f$. For such general case, this can be analyzed through decision making under uncertainty [34]. In our case, we consider our simulation field as a cluster composed of a circumference contained in a rectangular area as displayed in Fig. 1. We will assume that all sensors covering this area will experience the same $P_d$ and $P_f$.

In order to fulfill these requirements, the perceived SNR of each node (when detecting the PU signal) must be nearly the same. This is achieved when the distance from PU to FC complies with [14]:

$$R_{pu} \geq \frac{10^{0.10\theta} + 1}{10^{0.10\theta} - 1} R_c,$$ (7)

where $R_c$ is the cluster radius, and $\theta = 3$ is the path loss exponent suggested by Hata model [14].
In contrast to reported static solutions, the proposed stochastic optimization problem in (6) allows to deal with the dynamics of nodes movement given by the random variable \( d_j \). The static solutions are only applicable when \( d_j \) is a deterministic quantity, and consequently, they are implemented in a time-slotted approach, where \( d_j \) is assumed to be constant. However, this will imply that nodes should update their positions to compute the optimal solution on each timeslot. In consequence, it will account for an increased energy consumption regarding the last term in (5). On the other hand, in our proposal (to be discussed in the next Section) we devise a solution avoiding the regular updates of the node position by relying on the statistical description of the random variable \( d_j \) instead.

**IV. PROPOSAL TO MINIMIZE ENERGY IN CSS**

Stochastic optimization problems can be addressed by “wait-and-see” and “here-and-now” approaches [26]. The first approach provides a solution on each time-slot based on the current location of nodes, while the second one computes just a long-term solution based on the statistic of the movement of nodes. “Wait-and-see” finds the exact solution on each timeslot [28], while “here-and-now” finds a solution avoiding a repetitive execution of a given static algorithm. This feature makes “here-and-now” approach a preferred candidate to reduce the total energy consumption values.

The problem exposed in (6) has been already addressed by equivalent formulations relaying on the first- and second-order moments of \( \tilde{E}_T \) in [29]. However, this will result in an inaccurate energy behavior provided that insufficient statistical features of the random variable are analyzed. Instead, here we formulate an equivalent problem statement by considering the cumulative distribution function regarding the network energy consumption, which in turn will provide a more accurate description. Based on this approach, we establish an upper bound on the consumed energy by the network, then we minimize this upper bound, denoted by \( E_\varphi \), considering the detection performance following the Kataoka criterion as [27]:

\[
\begin{align*}
\min_{\rho_j} \quad & E_\varphi \\
\text{s.t.} \quad & \mathbb{P}(\tilde{E}_T(\rho_j, \tilde{d}_j) \leq E_\varphi) = \theta \\
& 1 - \prod_{j=1}^{N} (1 - \rho_j P_{f_j}) \leq \alpha \\
& 1 - \prod_{j=1}^{N} (1 - \rho_j P_{d_j}) \geq \beta \\
& \rho_j \in \{0, 1\}, \quad E_\varphi > 0,
\end{align*}
\]

where \( \theta \in [0,1] \) denotes the probability that the random variable \( \tilde{E}_T(\rho_j, \tilde{d}_j) \) will be upper bounded by the quantity \( E_\varphi \). This formulation aims to find the lowest energy consumption level \( E_\varphi \), while simultaneously considering a given performance by the global false alarm and detection thresholds \( \alpha \) and \( \beta \), respectively.

To solve the problem in (8), we first re-state the problem formulation by clearing \( E_\varphi \) from (8a) and \( \rho_j \) from (8b). Besides, we will assume that \( \rho_j \) is a continuous parameter on the interval \([0,1]\). All together will establish an equivalent problem formulation similar to the one reported in [13], where approximate solutions are derived for a complex-reduced formulation in contrast to the NP-complete hard-case in (8).

To clear the variable \( E_\varphi \) from (8a) we re-write this restriction in a more tractable expression by using vector notation. In addition, provided the proposed method does not update nodes position on each time-slot, then \( E_{P_f} = 0 \). We state the energy formulation defined above in vector form as \( \tilde{E}_T = \mathbf{E}_1^t \mathbf{p} + \delta \mathbf{E}_2^t \mathbf{p} \), where \( \mathbf{E}_1 = [E_{s_j} + E_{v_{elec}}] \), \( \mathbf{E}_2 = [\mathbf{e}_{amp}] \), \( \mathbf{p} = [\mathbf{p}_j] \), and the superscript \( t \) denotes the transpose operation. By this way, the condition in (8a) is simplified as follows:

\[
\mathbb{P}(\mathbf{E}_1^t \mathbf{p} + \delta \mathbf{E}_2^t \mathbf{p} \leq E_\varphi) = \mathbb{P}\left( \frac{\mathbf{E}_1^t \mathbf{p}}{\mathbf{E}_2^t \mathbf{p}} \leq \frac{E_\varphi}{\mathbf{E}_2^t \mathbf{p}} \right) = \mathcal{F}_{\mathbf{e}_1}(\mathbf{E}_2^t \mathbf{p}) = \theta,
\]

where \( \mathcal{F}_{\mathbf{e}_1}(\cdot) \) represents the cumulative distribution function of squared distance from each sensor node to FC. The function \( \mathcal{F}_{\mathbf{e}_1}(\cdot) \) will be dependent on the mobility pattern regarding the movement of nodes. Their obtaining for three different mobility patterns (Random Walk, Random Waypoint, and Gaussian-Markov) will be illustrated in Section VI. This function can be previously derived and stored according to particular mobility patterns of sensor nodes, then avoiding any complexity load in the online operation. Solving the equation (9) for \( E_\varphi \), then we obtain:

\[
E_\varphi = \mathbf{E}_1^t \mathbf{p} + \mathcal{F}_{\mathbf{e}_1}^{-1}(\theta) \mathbf{E}_2^t \mathbf{p},
\]

where upon substitution in (8) we obtain and equivalent objective function to be minimized.

Additionally, we rewrite the global probability of false alarm constraint in (8b) by clearing \( \rho_j \). Considering that \( P_{f_j} \), as defined in (1), has the same value for each sensor node, and upon substituting (1) into (8b), we rearrange terms and apply the logarithm function on both sides as:

\[
\ln(1-\alpha) \leq \sum_{j=1}^{N} \ln \left( 1 - \rho_j Q \left( \frac{\varepsilon}{\sigma_{s_j}} - 1 \sqrt{\delta f_s} \right) \right)
\]

By this resulting operation, the product is transformed into a sum, and after simplifying we can obtain an upper limit \( M \), referred to the total number of active sensor nodes by means of the floor function. Thus, the modified constraint (8b) can be expressed as follows [13]:

\[
\sum_{j=1}^{N} \rho_j \leq \frac{\ln(1-\alpha)}{\ln \left( 1 - Q \left( \frac{\varepsilon}{\sigma_{s_j}} - 1 \sqrt{\delta f_s} \right) \right)} = M.
\]
Finally, by replacing the derived objective function (10) (after solving the vector operations), as well as the by replacing (12) in (8), and considering that \( \rho_j \) is a continuous variable, we obtain a reduced-complex problem formulation as follows:

\[
\min_{\rho_j} \sum_{j=1}^{N} \rho_j \left( E_{s_j} + E_{t-elec} + e_{\text{amp}} F_{d_j}^{-1}(\theta) \right) \tag{13}
\]

s.t.
\[
\sum_{j=1}^{N} \rho_j \leq \frac{\ln(1 - \alpha)}{\ln(1 - Q(\frac{\beta - \sqrt{\delta_{\nu}}}{\sigma_j} - 1)\sqrt{\delta_{\nu}})} = M \tag{13a}
\]

\[
1 - \prod_{j=1}^{N} (1 - \rho_j P_{d_j}) \geq \beta \tag{13b}
\]

\[
\rho_j \in [0, 1]. \tag{13c}
\]

This constrained problem can be reformulated by means of the method of Lagrange multipliers to convert it into an unconstrained problem. The Lagrangian function is expressed as a function of the decision variable \( \rho_j \), and the undetermined Lagrange multipliers \( \lambda \) and \( \eta \) for (13a) and (13b) constraints, respectively, as follows:

\[
L(\rho_j, \lambda, \eta) = \sum_{j=1}^{N} \rho_j \left( E_{s_j} + E_{t-elec} + e_{\text{amp}} F_{d_j}^{-1}(\theta) \right) + \eta \sum_{j=1}^{N} (\rho_j - M) - \lambda \left( 1 - \prod_{j=1}^{N} (1 - \rho_j P_{d_j}) - \beta \right). \tag{14}
\]

To determine the optimal solution, it is required to analyze the first-order partial derivative conditions to find the stationary points. However, this implies solving a system of equations of \( N \) unknown \( \rho_j \) variables which in turn becomes computationally expensive. To circumvent this issue, we define a cost function to account for the total consumed energy and detection performance constraints similar to the approach in [13]. This cost function-based approach aims to obtain a sub-optimal number of active nodes by reducing the computational complexity inherent to the problem posed in (13). The evaluation of the cost function will provide the preferred nodes to participate in CSS to account for a reduced energy consumption result.

In this case, the cost function per \( j \)-th sensor node is expressed as follows:

\[
C_j = E_{s_j} + E_{t-elec} + e_{\text{amp}} F_{d_j}^{-1}(\theta) - \lambda P_{d_j}. \tag{15}
\]

This priority metric is derived through the first-order partial derivative condition regarding the Lagrangian function in accordance with the optimization problem in (13). Nodes with the lowest \( C_j \) will be selected, that is, those nodes with the lowest energy-parameter values and highest probability of detecting the PU signal will represent the best candidates to run the spectrum sensing operations.

In order to guarantee the optimality of the proposed approach based on cost functions, we must examine the Karush Kuhn Kuhnert conditions. Similarly to [13], it is mandatory to ensure that the global probability of detection inequality \( P_d \geq \beta \) and the modified constraint regarding the probability of false alarm \( \sum_{j=1}^{N} \rho_j \leq M \) are achieved, simultaneously, this to satisfy the complementary slackness conditions. Therefore, the heuristic algorithm (to be discussed in Section V) shall turn on \( M \) sensor nodes in the worst-case scenario to fulfill the previous statements. Thus, the proposed solution will return an optimum value for the decision variable \( \rho_j \) to minimize the wasted energy in spectrum sensing operations.

Summarizing, to solve the problem in (13) we evaluate the cost function for each node in (15). Then nodes are ordered considering the resulting cost function value. Based on this ordered array, nodes with lower cost function value will determine the preferred nodes to participate in CSS, while the remaining nodes will operate on sleep mode to save energy. The total number of active nodes will be obtained by the minimum set of ordered nodes to accomplish the global probability of detection \( P_d \). This resulting total number of nodes will be also upper-bounded by \( M \) in (13a) in order to not exceeding the global probability of false alarm \( P_f \). Finally, to consider the implementation of this solution, these steps have to be implemented iteratively to find a feasible solution to the proposed optimization problem. This will be introduced in Section V.

**A. FURTHER ANALYSIS ON KATAOKA CRITERION**

The total number of awake sensor nodes and their corresponding consumed energy are derived according to the cost function introduced in (15). This priority metric depends upon the inverse cumulative distribution function \( F_{d_j}^{-1}(\theta) \), the Lagrange multiplier \( \lambda \), and the probability of detection \( P_{d_j} \). To evaluate this cost function, we have to obtain in advance the inverse cumulative distribution functions by numerical methods, then to establish some assumptions regarding the homogeneity of detection capabilities to each sensor node.

The inverse cumulative distribution function is directly related to mobility models. Thus, we have to find \( F_{d_j}^{-1}(\theta) \) for the movement pattern that matches the dynamics of sensor nodes. To that end, nodes mobility models must be simulated by a wide time-interval, then we can derive the cumulative distribution function via a given histogram of \( F_{d_j} \). This histogram is obtained by running simulations to compute the distance from each sensor node position to FC versus sensor nodes’ probability of occurrence. To derive an analytic model, we fit the inverse of the obtained histogram by a polynomial using numerical methods. Goodness-of-fit statistics must be guaranteed by a performance metric such as the coefficient of determination (R-squared).

Besides, we assume that every sensor node experiences the same SNR value (as explained before) to account for a constant \( P_d \) value. This assumption allows to simplify the
cost function in (15) by omitting the last term as:

\[ C_j = E_{s_j} + E_{t\text{-elec}} + e_{\text{amp}} F_d^{-1} (\theta), \]  

(16)

provided this constraint quantity between nodes does not bring any selection criteria after ordering.

Then, the priority metric to determine awake sensor nodes is reduced to evaluate energy parameters as shown in (16). The minimum number of awake sensor nodes will be given by those sensor nodes with lower cost function value in (16) and simultaneously ensuring detection performance. Total energy consumption will be computed by evaluating the objective function in (13) considering solution vector \( \rho \). Next section is devoted to present an algorithm based on this strategy.

**V. ITERATIVE ALGORITHM**

The iterative algorithm to select the total number of sensor nodes involved in CSS follows the addressed solution in Section IV. This algorithm applies to the Kataoka criterion considering the reduced cost function in (16). Its implementation is presented in Algorithm 1 to find the minimum total number of awake sensor nodes to satisfy detection performance, then to reduce consumed energy.

**Algorithm 1 Iterative Algorithm**

**Inputs:** \( N, s \)  
**Outputs:** \( \rho, E_T \)

1. Initialize \( \text{nodes} = 1 \) and \( \rho \) as a 1-by-\( N \) array of zeros
2. Fit function \( F_d^{-1} (\theta) \) for the movement model used
3. Compute \( P_D \) and \( M \) based on (2) and (13a)
4. for \( j = 1 \) to \( N \) do
5. Calculate appropriate cost function \( C_j \) based on (16)
6. end for
7. Rearrange \( C_j \) in ascending order and store corresponding indexes in array \( \text{cost\_ordered} \)
8. while \( \text{nodes} \leq M \) do
9. \( \text{selected\_nodes}(\text{nodes}) = \text{cost\_ordered}(\text{nodes}) \)
10. \( \rho(\text{selected\_nodes}(\text{nodes})) = 1 \)
11. Calculate \( P_D \) with selected sensor nodes from \( \text{selected\_nodes} \) array based on (4)
12. if \( P_D \geq \beta \) then
13. break
14. end if
15. if \( \text{nodes} = M \) then
16. \( \rho = 0 \)
17. break ("\( P_D \) is not ensured")
18. end if
19. \( \text{nodes} = \text{nodes} + 1 \)
20. end while
21. Compute \( E_T \) based on energy objective function in (13)
22. return \( \rho, E_T \)

Algorithm 1 is mainly composed of two sections: preprocessing phase, from line 1 to 7, and sensor selection phase, from line 8 to 20. Preprocessing phase performs three tasks such as initializing required variables (line 1), fitting and evaluating the inverse cumulative distribution function \( F_d^{-1} (\theta) \) for corresponding mobility model (line 2), and finally, computing and rearranging cost function values (lines 3 to 7). The sensor selection phase is composed of a while loop, from line 8 to 20, to determine which nodes will participate in CSS and which ones shall go to sleep mode to extend energy batteries.

The while loop (lines 8 to 20) returns the solution given by the vector \( \rho \) represented on the variable \( \text{selected\_nodes} \). \( P_D \) is computed after iteratively including nodes (line 9 and 10) with lowest cost function values in line 11. This inclusion ends when detection performance is achieved (lines 12 to 14) or when the total number of included nodes exceeds the upper limit \( M \) by testing this condition on lines 15 to 18. Selected nodes are the ones to be activated by asserting the proper elements of vector \( \rho \) based on the obtained array \( \text{selected\_nodes} \) by the following rule in line 10. On each loop iteration, variable \( \text{nodes} \) is incremented by 1 as stated in line 19 until it reaches the upper limit \( M \). The last step is to return awake sensor nodes specified in the variable \( \rho \) and the energy metric stored in the variable \( E_T \) as established in line 21.

The convergence of the proposed algorithm is analyzed concerning the bounds on the total number of iterations before the desired outputs are reached [35]. In specifics, the optimality conditions previously discussed impose that the iterative algorithm must be upper bounded by \( M \) to guarantee the global probability of false alarm constraint. That means, the main while loop of the algorithm, from line 8 to 20, will be executed \( M \) times in the worst-case scenario, otherwise, a feasible solution is not ensured because the \( P_F \) constraint in (13a) is violated.

To consider complexity of Algorithm 1, main while loop is dependent on parameter \( M \), which is upper bounded by \( N \). Moreover, a nested loop is executed in line 11 to compute the global probability of detection \( P_D \) based on \( \text{selected\_nodes} \) array. Hence, the computational complexity of the proposed algorithm is \( O(N^2) \) improving the exhaustive search algorithm of complexity \( O(N!) \).

**VI. CASE OF STUDY**

To illustrate, we consider that sensor nodes are moving randomly over the simulation area following specific random patterns. There are several mobility models applicable to wireless networks divided into two main groups: entity and group mobility models [36], [37]. Entity models are focused on individual movements of sensor nodes while group models describe displacements depending on the position of remaining nodes.

We assume our system model is composed of nodes describing entity models due to the lack of group mobility rules. These models are implemented on open-source Java software called BonnMotion, which generates various mobility scenarios [38]. In this study, we use three different mobility models from lowest to highest precision to describe realistic...
movement patterns such as Random Walk, Random Waypoint, and Gauss-Markov [39]–[41]. The behavior of nodes on the borders is assumed by the rules of the software BonnMotion, where nodes that reach the edges of the simulation field will bounce off the border with an angle determined by the incoming direction. These mobility models are selected looking for the development of suitable solutions applicable to current LTE femto-cells mobile networks as described in [42]–[44], and envisaging future 5G and beyond systems. The main features of these entity models are exposed below.

Random Walk model describes unpredictable behavior in nature. Each sensor node moves from its current location to another position by randomly choosing a new direction and speed. These parameters are maintained until they travel a certain distance or a fixed time has elapsed. The speed and direction are established following predefined ranges given by \([s_{\text{min}}, s_{\text{max}}]\) and \([0, 2\pi]\), respectively. Fig. 2 (a) and (d) show an example of a sensor node moving within a bounded square area of side 100 m, and corresponding probability density function, respectively. We have selected the distance-constrained method for this random walk simulation which in turn establishes an equal distance movement model. Also, the Random Walk model is a memory-less system. This feature results in unrealistic patterns such as sharp turns that may be incompatible with practical scenarios.

Random Waypoint model includes pause times between changes in speed and direction. The remaining characteristics are similar to the random walk model. Sensor nodes travel from one location to another for a fixed time or until a given distance is reached. After that, nodes stay in the current position until the pause time expires and choose a new speed and direction. Here, Random Waypoint is similar to Random Walk when pause time is equal to zero. To illustrate, Fig. 2 (b) and (e) depict displacements of a given sensor node over a bounded square area by using the time-constrained method, and the normalized histogram of squared distance to FC, respectively. Discrete steps have variable distance values in contrast to the model shown in Fig. 2 (a).

Gauss-Markov mobility model represents a memory system able to adapt to different levels of randomness via one tuning parameter \(v \in [0, 1]\). Initially, we assign the speed and direction of nodes to travel a fixed time \(n\). Speed and direction parameters at the \(n\)-th step are defined by \(s_n = vs_{n-1} + (1 - v)s\) and \(d_n = vd_{n-1} + (1 - v)d\), respectively. Parameters \(s\) and \(d\) represent the mean value of speed and direction when \(n\) tends to infinite and parameters \(s_{n-1}\) and \(d_{n-1}\) are Gaussian random variables. Finally, next location for each sensor node is computed based on current location, speed, and direction as:

\[
x_n = x_{n-1} + s_{n-1} \cos (d_{n-1}), \quad (17)
\]

\[
y_n = y_{n-1} + s_{n-1} \sin (d_{n-1}), \quad (18)
\]

where terms \((x_n, y_n)\) and \((x_{n-1}, y_{n-1})\) represent Cartesian coordinates at instance \(n\)-th and \((n - 1)\)-th, respectively.

Fig. 2 (c) illustrates behavior of a sensor node on a depicted square area by setting \(v = 0.75\), and Fig. 2 (f) shows its corresponding probability density function (numerically estimated by means of a histogram).

Based on these three models, we may simulate results in a broad sense by considering different conditions and patterns regarding the movement of nodes. To illustrate, we execute the proposed Algorithm 1 to obtain the total number of awake nodes and expended energy in CSS. Also, we implement the exhaustive search algorithm to study the gap between both solutions from the perspective of accuracy. Simulation scenario consists of a square field of side 100 m, and 20 sensor nodes participating in the network and performing random patterns. The inverse cumulative distribution function is fitted by a sixth-degree polynomial using numerical methods and evaluated by a given probability \(\theta = 0.9\). The goodness-of-fit statistic is guaranteed by an R-squared metric equals to 0.9999. Energy parameters values are established according to Chipcon transceiver datasheets [45], and will be detailed in Section VII. Detection constraints are specified as \(\beta = 0.9\) and \(\alpha = 0.1\).

Obtained results are summarized in Table 1 related to the total number of awake nodes, consumed energy, and performance given by the probability of detection and false alarm. A feasible solution is affordable on each mobility model provided that the detection constraints regarding \(P_D \geq 0.9\) and \(P_F \leq 0.1\) are simultaneously fulfilled. The proposed solution depicts an identical total number of awake nodes, given by \(\sum \rho_j = 11\), for each mobility model due to the assumption of a constant \(P_F\) value. Besides, expended energy \(E_T\) changes by the inverse cumulative distribution function of squared distance to FC. In this case, similar behavior of histograms, exposed in Fig. 2 (d) to (f), leads to similar values of energy consumption \(E_T\) for each mobility model.

To evaluate the suitability to address a non-convex optimization problem, we compare the outputs from Algorithm (1) with those of our proposal but solved with the exhaustive search algorithm. The total number of active nodes, the expended energy, and the global probability of false alarm of the proposed algorithm are about 10%, 17%, and 0.3% higher than the exhaustive search algorithm, respectively. However, the global probability of detection achieved with the Kataoka criterion is improved by 1% compared to the exhaustive search algorithm. Finally, we must point out that the exhaustive search algorithm examines \(\sum_{j=1}^{N} \binom{N}{j}\) combinations, which becomes computationally prohibitive for the central processing unit of sensor nodes.

VII. NUMERICAL RESULTS

In this section, we evaluate the performance of the proposed stochastic solution following “here-and-now” approach under the Kataoka criterion. With the aim of comparison, we take as reference “wait-and-see” approach based on Energy Efficient Sensor Selection (EESS) static solution as reported in [13]. Besides, we also consider the dynamic solution with the “here-and-now” approach based on the Expected Value
Standard Deviation criterion, which relies on Chebyshev’s inequality, to deal with the random position of sensor nodes as reported in [29]. It must be noted that “wait-and-see” approach requires sending the updated positions of each sensor node to the FC on each time-slot, to decide the node operation mode: awake or asleep. Indeed, for this approach, we do not contemplate the expended energy to compute the optimal solution on each time-slot provided it will be negligible in comparison to the energy expended to transmit the updated position to the FC.

We obtain the derived total number of awake sensor nodes and the corresponding consumed energy for each exposed mobility model. The simulation scenarios are analyzed for different network sizes and PU positions to evaluate their impact on detection performance.

For the proposed solution we evaluate the inverse cumulative distribution function $F_{d_i}^{-1}(\theta)$ in $\theta = 0.9$ (remark that the value of $\theta$ represents the probability of cumulative distances from each sensor node to the FC). Based on each specific model presented in Section VI and Fig. 2 (d) to (f), the inverse cumulative distribution function will return similar distance values for the three selected models. Besides, the Expected Value Standard Deviation criterion is implemented with parameter $k = 10$, this to guarantee that energy values less than $k$-times the standard deviations away from the mean will have the same probability of $(1 - \frac{1}{k}) = 0.9$, under the Chebyshev’s inequality. This seeks to establish a fair comparison with the Kataoka criterion to evaluate the same probability $\theta = 0.9$ in (8a).

We assume that our simulation field is inscribed in a circular cluster of radius $R_c = \frac{\sqrt{2}}{s}$ to guarantee that each sensor node has equal SNR value in accordance with (7). The PU is located outside the cluster satisfying the inequality shown in (7) and FC is placed at the center of the field. For simplicity, the used free-space propagation model comprises isotropic antennas, for which $G_T = 1$, $G_R = 1$ and $f_c = 2.4$ GHz.

### TABLE 1. Performance comparisons.

<table>
<thead>
<tr>
<th>Mobility Models</th>
<th>$\sum E_T \mu J$</th>
<th>$P_D$</th>
<th>$P_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kataoka Exhaustive Search</td>
<td>Kataoka Exhaustive Search</td>
<td>Kataoka Exhaustive Search</td>
<td>Kataoka Exhaustive Search</td>
</tr>
<tr>
<td>Random Walk</td>
<td>11</td>
<td>9</td>
<td>4.1372</td>
</tr>
<tr>
<td>Random Waypoint</td>
<td>11</td>
<td>9</td>
<td>4.0348</td>
</tr>
<tr>
<td>Gauss-Markov</td>
<td>11</td>
<td>9</td>
<td>4.1570</td>
</tr>
</tbody>
</table>
Detection thresholds are specified as $\alpha = 0.1$ and $\beta = 0.9$.

Energy consumption values are derived based on several models of Chipcon transceivers such as CC2400, CC2420, CC2430, and CC2500 [45]. The energy parameter $E_{s,j}$ in (15) is determined by adding two terms: a typical value of 40 nJ used for the power of the receiving electronic, and consumed energy values in signal processing phase of 122 nJ, 147 nJ, 200 nJ, or 153 nJ, depending on the appropriate transceiver. For instance, according to technical specifications of CC2500 transceiver, we compute the consumed energy related to signal processing for a data rate of 250 kb/s, a voltage of 1.8 V, and a current of 21.2 mA. This operation gives approximately 153 nJ/bit and we only use one bit per decision. Remaining energy parameters of the network are defined by $E_{t_{-elec}} = 80$ nJ and $e_{amp} = 40.4$ pJ/m$^2$ similar to [46]. It is important to emphasize that “wait-and-see” method must update the spatial location of nodes to compute the optimal solution. This will imply that additional $n$-bits of information will be transmitted from each sensor node to the FC on each time-slot to send its location. To illustrate, here we assume a precision of $n = 8$ bits for the $x$ and $y$ axes as depicted in Fig. 1.

Fig. 3 exhibits the consumed energy for a simulation field ranged from 50 m to 300 m. We plot the averaged output from the static EESS algorithm by considering 5000 time-slots, labelled as ‘avg eess’. The dynamic Expected Value Standard Deviation criterion, based on “here-and-now” and Chebyshev approaches, and labelled as ‘mean_std’, is also plotted. Based on this figure, the proposed solution based on the Kataoka criterion (labelled as ‘kataoka’) spends less energy to operate than the two other approaches: the “wait-and-see” based on EESS solution and the Chebyshev-based Expected Value Standard Deviation.

This favorable result is because of two major reasons: our solution is computed only once, and it is implemented based on the cumulative distribution function instead of only the first and second moments of the random variable. Although optimal (when computing the lower total number of active sensors), the “wait-and-see” approach has to spend extra energy when updating the position of nodes on each time-slot, which in turn will exhibit higher total energy as depicted in Fig. 3. On the other hand, the Chebyshev approach will be less accurate than the proposed method, and this will imply that its resulting total number of active nodes will be higher than the proposed method. Furthermore, both “here-and-now” approaches exhibit an increasing monotonic tendency of the energy consumption regarding the network size. This increasing slope is a consequence of the higher values reached by $\mathbb{P}_{\delta_j^-(\theta)}$ in (16) for bigger simulation areas.

Fig. 4 depicts the behavior of energy consumption for different PU positions, which in turn will imply a varying condition for the SNR parameter and the perceived local probability of detection in (2). This scenario considers a network side of 100 m and 100 participating sensor nodes. The particular PU position has a significant impact on energy values for the three algorithms and it shows a low effect due to the different mobility models as shown in Fig. 4. The farther away the PU is, the higher the energy consumption because local probabilities of detection will decrease. The proposed solution shows a lower gradient than “wait-and-see” EESS approach due to the increase of expended energy in updating the spatial location of nodes is significantly higher than just to send the local spectrum sensing results. Regarding the Expected Value Standard Deviation approach, the Kataoka solution reduces energy, making a better estimation of energy involved in CSS for the same probability of 90%.

Based on the obtained results and plots in Fig. 3 and 4, particular mobility models show similar behavior on consumed energy values. There is not any preferred mobility model to have a better performance metric. The specifics
of a given mobility model modify only consumed energy values through the term $F^{-1}_D(\theta)$ when evaluating the objective function in (13). This influence is negligible provided the similarities between the plotted probability density functions in Fig. 2 (d) to (f). Also, the proposed method allows decreasing expended energy over CSS provided our solution is computed only once. On the contrary, “wait-and-see” approach needs to be continuously computed on each time-slot to have specific solutions for each particular set of node positions on the field.

To account for the global probabilities of false alarm and detection, $P_F$ and $P_D$, respectively, we consider a transmitted PU signal composed of a rectangular pulse train. The transmitted PU signal is contaminated with Additive White Gaussian Noise (AWGN) and SNR parameter is ranged on $[-5, 2]$ dB interval. The total number of samples per pulse is 10 times the window samples length of the energy detector, given by $10^6 f_s$, and the total number of PU signal samples is $10^6$. Monte Carlo simulation is performed to estimate false alarm and detection local probabilities to validate the detection performance of the proposed solution.

Using OR rule, we compute $P_D$ which has to be greater than detection threshold $\beta$ following constraint in (13b). Fig. 5 (a) shows the obtained $P_D$ vs SNR by solid line and detection threshold $\beta$ by dashed line. $P_D$ curve depicts a behavior compliant to constraint $P_D \geq \beta$. This curve is monotonically increasing on those intervals where the proposed method computes the same number of nodes. Local minimums describe such SNR values where the proposed solution reduces the total number of awake nodes. The global probability of detection always exceeds the detection threshold $\beta = 0.9$ to avoid interference with the PU signal and increase bandwidth.

Similarly, we determine $P_F$ which has to be lower than the false alarm threshold $\alpha$. Fig. 5 (b) depicts the $P_F$ curve by a solid line and false alarm threshold $\alpha$ by a dashed line. $P_F$ curve shows a decreasing slope as a result of fewer nodes have been selected to participate in the spectrum sensing phase. The global probability of false alarm never exceeds the false alarm threshold $\alpha = 0.1$ which in turn guarantees the false alarm constraint imposed in (13a).

FIGURE 5. Detection constraints for different SNR. a) Global probability of detection. b) Global probability of false alarm.

VIII. CONCLUSIONS

This paper addresses the dynamic behavior of nodes on CRSN applications by considering various mobility models for sensor nodes such as Random Walk, Random Waypoint, and Gauss-Markov. We introduce a novel framework to analyze the energy dynamic model to reduce energy consumption on CSS. To this end, a stochastic optimization problem was proposed to minimize energy consumption based on the Kataoka criterion. Thus, it drives to deal with the appropriate sensor selection in spectrum sensing for an accurate estimation of the network energy consumption. This particular allows us to implement a better node selection mechanism for dynamic CRSN. Although sub-optimal, the numerical results validate the achieved reduction on energy consumption values while fulfilling the performance of proposed solution. Besides, we can also conclude that movement models do not have a significant influence on energy consumption values, but they rather present similar behavior. Future work will be focused on a variety of directions as addressing the global probability of detection as a random variable provided the dynamics of sensor nodes, the implementation of evolutionary optimization solutions on dynamic environments, considering the internal status of the node’s battery, extending results to larger network sizes, and the inclusion of more specific factors as the PU traffic and throughput.

REFERENCES


[27] A. Hatoum, R. Langar, N. Aitsaadi, R. Boutaba, and G. Pujolle, “Cluster-


