

Centralized Model-Predictive Control with Human-Driver Interaction for Platooning

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Abstract—Cooperative adaptive cruise control presents an opportunity to improve road transportation through increase in road capacity and reduction in energy use and accidents. Clever design of control algorithms and communication systems is required to ensure that the vehicle platoon is stable and meets desired safety requirements. In this paper, we propose a centralized model predictive controller for a heterogeneous platoon of vehicles to reach a desired platoon velocity and individual inter-vehicle distances with driver-selected headway time. In our approach, we allow for interruption from a human driver in the platoon that temporarily takes control of their vehicle with the assumption that the driver will, at minimum, obey legal velocity limits and the physical performance constraints of their vehicle. The finite horizon cost function of our proposed platoon controller is inspired from the infinite horizon design. To the best of our knowledge, this is the first platoon controller that integrates human-driven vehicles. We illustrate the performance of our proposed design with a numerical study.

Index Terms—Platooning, Cooperative Adaptive Cruise Control, Model-Predictive Control, Human-Driver Interaction, Hybridized Cyber-Physical Systems.

I. INTRODUCTION

Autonomous vehicle platooning with inter-vehicle communication permits road vehicles to travel close together increasing road capacity while reducing energy use and associated vehicle emissions [1]. This cooperative connected cruise control technology can reduce the incidence of so-called ghost traffic jams [2] and highway accidents [3]. The autonomous cruise control problem was first posed as a centralized platoon design approach in [4], and has seen recent attention with several survey papers [2], [5], [6].

Adaptive Cruise Control (ACC) systems use on board sensors to measure the distance and velocity of a predecessor vehicle to operate an autonomous cruise control system. However, these systems are prone to string stability issues resulting in ghost traffic jams [7]. To ensure stability of the platoon, either a large, velocity-based inter-vehicle distance is required using a headway time [8] or more than just the preceding vehicle's state is required. By utilizing inter-vehicle communication systems, Cooperative Adaptive Cruise Control (CACC) systems can reduce the inter-vehicle distance and

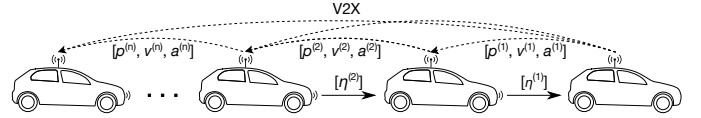


Figure 1. CACC concept: a platoon controller regulates the velocity and inter-vehicle distances utilizing the vehicles' position p , velocity v , and acceleration a , exchanged via inter-vehicle communications.

avoid string stability issues by sharing the desired control actions from other vehicles. Figure 1 illustrates a potential CACC design of a coordinated platoon of vehicles driving with small inter-vehicle distances with a platoon communication network.

Many platoon control designs have focused on the technical aspects of algorithm design, communication constraints, or experimentation isolated from real traffic [9]. However, studies of driver behavior have found that human drivers maintain small inter-vehicle distances below safety margins [10], even to less than one second when near autonomous platoons [11]. During implementation of vehicle platooning systems, it will be important to incorporate legacy vehicles that are unable to integrate with a platoon communication system and human-driver interaction for passenger comfort and well-being, e.g., some passengers feel uncomfortable with too small safety gaps, as well as motion sickness. This is in line with research integrating human behavior and Cyber-Physical System (CPS), moving from CPS to Hybridized Cyber-Physical System (H-CPS) which is also called Cyber-Physical-Social System (CPSS) [12].

In this paper, we present a novel constrained Model Predictive Control (MPC) approach for the centralized control of a platoon of heterogeneous vehicles with reconfiguration under temporary human-driver control. The desired inter-vehicle distance is based on driver-selected headway times which are variable between individual vehicles in the platoon, and can change over time [13]–[15]. Figure 2 illustrates the control loop of our proposed centralized platoon controller. The exogenous inputs to our controller are the desired platoon velocity and individual inter-vehicle distances, and the desired safety constraints on the inter-vehicle distance, velocity, and acceleration. The output from our controller is the desired control action for every vehicle in the platoon.

To ensure convergence of the vehicle positions and velocities to the desired references over a finite horizon, we design a time-varying reference for all vehicles in the platoon. We consider a Multiple-Input Multiple-output (MIMO) model

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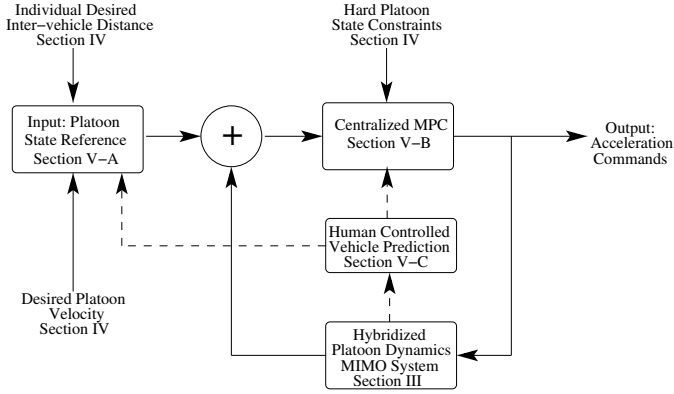


Figure 2. Platoon control architecture. The exogenous inputs to the controller are the desired inter-vehicle distances and platoon velocity to form the platoon reference. The output is the control action for all vehicles in the platoon. The controller reconfigures when a human-driver temporarily takes control. The block connected by the dashed lines engages predicting a minimum control action of the human-driver.

of the platoon of individual vehicle dynamics to predict the motion of the vehicles from the control inputs. Inspired by the infinite horizon optimal control algorithm [16], we propose a finite horizon cost function. Our cost function penalizes the quadratic error of the inter-vehicle distances to desired inter-vehicle distances, as well as the positions, velocities, and accelerations to our time-varying reference. The use of a quadratic cost function allows for simple application of hard constraints on the cost function in a quadratic program optimizer to guarantee minimum and maximum inter-vehicle distances, velocities, and accelerations of all vehicles in the platoon.

In our design, we include a switch to reconfigure our controller for a vehicle temporarily under human driver control, shown as the dashed line in Figure 2. Our approach accommodates a human driver to make an emergency brake, reduce speed, or travel at a different inter-vehicle distance, or could be utilized to incorporate legacy vehicles. The switch adjusts our time-varying reference to return the platoon to the desired reference and removes the vehicle from the platoon control while still incorporating the vehicle states such that the control actions for the platoon still ensure the inter-vehicle distance constraints. We predict the control actions of the human driver by assuming that the driver will only change their control action to obey a minimum set of constraints of the legal road speed limits and their vehicle performance constraints. More complex predictive approaches, such as economic cost functions [17] or inter-vehicle interactions, could be utilized to predict a human driver.

The key contributions of this paper are

- proposal of a constrained MPC algorithm for centralized platoon control of heterogeneous vehicles,
- with finite horizon quadratic cost function inspired by works in infinite horizon optimal control [16],
- inclusion of driver-selected headway time, and
- reconfiguration under temporary human-driver control.

II. RELATED WORK

In this section we review the relevant literature on CACC from the perspective of both communication systems and control design, as well as human driver integration, before formulating our problem.

A. CACC Communication

In order to work reliably and with small safety gaps, CACC requires periodic updates of vehicles' data (e.g., acceleration, speed, position). Typically, the data from at least the vehicle in front and often also the first vehicle (i.e., the platoon leader) is necessary. If the updates arrive with a high enough frequency for the control system to react properly, string-stability, i.e., keeping the desired gaps without accumulating control errors throughout platoon members, can be achieved [18]. If the updates are delayed, "string-stability is seriously compromised" [19].

1) *IEEE 802.11p*: Up to a few years ago, the main Vehicle-to-Everything (V2X) technology considered was IEEE 802.11p as a basis for quite advanced protocol families such as ETSI ITS-G5 [1].

The most simple approach for exchanging the vehicle updates is to use *static beaconing*, where vehicles broadcast their information in regular, periodic intervals. Yet, static beaconing can lead to a congested channel, especially in highly dense scenarios, e.g., with long or many platoons, thus reducing the stability of a platoon. Thus, Segata et al. [15] proposed to use *slotted beaconing*, which splits the time for the leader beacon into transmission slots for all platoons members. The authors show that this can greatly improve the beaconing performance in crowded scenarios, especially when combined with transmit power control, thus, reducing the load and improving the reliability.

In order to reduce the channel load further, *dynamic beaconing* schemes have been proposed. Sommer et al. [20] presented the Adaptive Traffic Beacon (ATB) protocol, which adaptively adjusts the beaconing period according to the current channel quality and the message utility. Following up on this, ETSI standardized Decentralized Congestion Control (DCC) [21]. It uses a simple finite state machine to adjust, among others, beacon interval and transmit power based on the current observed channel busy ratio. Sommer et al. [22] proposed *DynB* to avoid overloading the wireless channel and allow low-latency communication by using very short beaconing intervals. The protocol continuously observes the channel load and considers detailed radio shadowing effects, even by moving vehicles, that block the transmission and the number of neighboring vehicles to calculate the best beacon interval. Focusing specifically on platooning, Segata et al. [23] proposed a dynamic approach called *jerk beaconing*, which exploits vehicle dynamics to share data only when needed by the controller. Here, the beacon interval is computed dynamically based on changes in acceleration over time, i.e., the jerk. This approach shows huge benefits in terms of network resource saving and is able to keep inter-vehicle distance close to the desired gap even in highly demanding scenarios.

Going beyond IEEE 802.11p, Segata et al. [24] proposed the Distributed EDCA Bursting (DEB) protocol, which extends the frame busting mechanism of IEEE 802.11p such that only the platoon leaders contend for the channel. In case of successful channel reservation, all vehicles in the platoon transmit a coordinated burst of frames, thus, sharing the platoon leader's transmission opportunity. This helps overcoming channel limits by reducing the number of nodes contending for the channel and improving spatial reuse. Amjad et al. [25] extend IEEE 802.11p by adding a full-duplex relaying system, which enables platoon members to simultaneously receive and relay the leader beacons.

2) *Cellular V2X (C-V2X)*: Albeit all of the above protocols and modifications, IEEE 802.11p alone seems not to be sufficient for meeting the strict requirements of CACC (i.e., ultra-low reliability and latency) [26]. The most prominent alternative for enabling V2X communication is C-V2X, which uses 3GPP standardized 5G cellular networks. Radio resources are scheduled by either the base station if vehicles are in-coverage (operation mode 3) or by a distributed resource allocation scheme if vehicles are out-of-coverage (operation mode 4). The latter allows vehicles to select resources in a stand-alone fashion with *semi-persistent scheduling*. While mode 3 in general allows for high packet reception ratios, mode 4 produces lower beacon update delays [27], which are also required for platooning.

For example, Vukadinovic et al. [28] compare IEEE 802.11p to 3GPP C-V2X based on LTE in both operation modes for truck platooning. Results show that C-V2X in both modes allows for shorter inter-truck distances than IEEE 802.11p due to more reliable communication in a congested wireless channel. However, short communication distances and large vehicle densities seem to be covered better with IEEE 802.11p instead of C-V2X [29]. Therefore, general modifications for improving the scheduling of sidelink radio resources in mode 4 have been proposed [30], [31]. In order to reach the performance required for CACC, Hegde et al. [32] propose to schedule the sidelink radio resources for the platoon members by the platoon leader. Similarly, the radio resource coordination method by Campolo et al. [33] fulfills the ultra-low latency requirements of CACC and is able to provide spatial reuse of LTE resources among platoon members.

3) *RADCOM*: Complementary to IEEE 802.11p and C-V2X, joint communication and sensing approaches, also known as Radar-based Communication (RADCOM), have been proposed. Following the trend of using higher communication frequencies for radio communication, Millimeter Wave (mmWave) technologies have recently become interesting to the V2X research community. mmWave technology promises high bitrates and low delays due to its wide channel bandwidth and dynamic beam-forming [34]. However, using it as a single communication technology may be difficult due to its highly volatile transmission channel, especially in an automotive environment [34], [35]. Nevertheless, initial works indicate that mmWave can be very valuable when complementing the other alternatives [36], [37].

B. CACC Controller

Control design for vehicle platooning has focused on meeting string stability conditions with several definitions in the literature [38]. In addition to stability requirements, the control design also needs to consider the information flow topology arising from the available communication links, formation geometry or spacing policy, vehicle dynamics, and desired platoon convergence.

The information flow topology of how information is shared between vehicles influences both the control algorithm design and the required communication system. Many control designs utilize a leader-follower approach where a lead vehicle sets the platoon speed and each follower vehicle maintains their own spacing to the predecessor, such as the sliding mode controller in [39] and employed in [15]. Other designs consider bi-directional information sharing from the neighboring vehicles such that leader information is not required, e.g. [40] and [41]. These distributed approaches consider that the lead or reference vehicle is exogenous to the platoon controller [42], either controlled by a human driver or by a separate ACC system [15]. Many designs focus on Vehicle-to-Vehicle (V2V), also of interest are V2X where infrastructure can monitor and coordinate a platoon [5], as well as interactions with other platoons [43]. Recent works have included unreliable communication channels in the control design using time delays [44] and packet loss [45].

A key design factor is the formation geometry of the inter-vehicle distances [8], which was considered fixed in early works [4] but caused string instability for ACC [38]. To achieve string stability for ACC, [46] proposed a velocity based spacing policy following the concept that a human driver should follow a preceding car with a certain headway time, with refinements in [47] and [48]. To account for the slower braking performance of heavy vehicles, a variable spacing policy with the headway a function of the difference in velocity [49]. The authors note that if lead vehicle information is shared, then the headway is able to be reduced to zero [50]. Often a common (non-unique) and constant headway time is utilized [51]. Alternative spacing policies have also included use of the traffic density [52], as constant time headway can result in unstable traffic flow [53].

The vehicle dynamics used for control design of vehicle platoons have included complex models that model torque output of the engine with variable gear ratios as well as simplified linear models. In reference to the nonlinear engine and gearbox models, it is noted in [39] that a first order lag model is suitable for higher level control of the vehicle, such as for platooning applications. This simplifies the vehicle, engine, and braking systems into a single constant. In [15], [39] the mechanical lag coefficient for a standard passenger vehicle is assumed to be $\tau_i = 0.5$ [s], with a heavy vehicle having a larger coefficient. Alternative modeling approaches have used the energy based port-Hamiltonian system model [40], [54]. While use of a homogeneous platoon with identical dynamics makes the control design and tuning simpler, it is unrealistic to real world heterogeneous platoons of different vehicles [55]. Certain controller stability properties can change with different

types of vehicles such as platoons of heavy vehicles [56], and environment effects including changes in road slope [57] and wind [55].

A variety of control design approaches have been proposed in the literature including the classic Linear Quadratic Regulator (LQR) [4], [41], [58], Proportional Integral Derivative [59], H-infinity [55], sliding mode control [39], and MPC. MPC algorithms optimize a finite horizon cost function at each time step, and allow for the inclusion of hard constraints [60], such as road speed limits and minimum safe inter-vehicle distances. The desired control actions from a constrained MPC controller will not exceed a vehicle's performance limit or control vehicles into situations that could lead to an accident. Additionally, so-called economic MPC [61], that assigns real values, such as fuel costs, to the weights in the cost function, has been utilized [62] to link vehicle performance to an energy or financial metric.

Distributed MPC algorithms in the leader-follower approach have been applied to platoons with poor communications [42], with extension to heterogeneous platoons [63], and string stability was enforced using constraints [64]. A more complex approach was employed in [65] to include network information as a delay on the desired control action in the dynamics.

Most control designs for the platooning of vehicles consider a distributed approach with the use of a lead or ego vehicle that provides an input reference to the platoon. This is a flexible approach as it allows for control designs to break apart and reform platoons [5]. However, distributed policies have been introduced that slow front vehicles and speed up later vehicles to form a platoon, while observing that this may be in conflict with the lead driver's goal of reaching their destination quickly [66]. Additionally, [49] noted that the use of a variable headway and ACC controller introduced a "group conscience" such that the leading vehicles were designed with reduced performance to take into account later vehicles in the platoon. A centralized control design would utilize all platoon information and a platoon reference to design the control actions for the vehicles as a collection.

The original work on the control of vehicular platoons is [4]. The authors designed a centralized LQR controller that took a target reference velocity for the platoon and desired inter-vehicle spacing, to generate the control action for all vehicles in the platoon, which was furthered in [67]. However, in [16] it was shown that the original cost function in [4] is not string stable as the length of the platoon goes to infinity, such that as more vehicles are added the convergence time expands, and the initial control action increases. The authors posed an alternative state representation and cost function that penalized both the absolute position error to the reference as well as the inter-vehicle distances to achieve finite convergence [16].

However, this approach is criticized in [68] which shows that an infinite length platoon is not equivalent to a large but finite platoon. In [69] it is shown that the optimal control design fails for certain initial conditions with large control values resulting from the static gain computed from the LQR such that desired control action could be larger than the maximum allowable control. The poor performance of large platoons also occurs in decentralized designs where the state

feedback control gain reduced for vehicles further away [70]. This reduction in state feedback gain was used to argue that for an M length vehicle platoon, there should be M independent controllers with M separately tuned gains [71].

III. PLATOON ARCHITECTURE

In this section, we state the single vehicle dynamics and develop the centralized platoon MIMO model of heterogeneous vehicles. We then state our human driver model.

A. Vehicle Dynamics

Consider the commonly utilized linear dynamics for longitudinal motion of a vehicle- i from [39] of $\dot{p}^{(i)} = v^{(i)}$, $\dot{v}^{(i)} = a^{(i)}$, and

$$\dot{a}^{(i)} = -\frac{1}{\tau_i}a^{(i)} + \frac{1}{\tau_i}u^{(i)}$$

where $p^{(i)}$ [m] is a point at the front bumper, $v^{(i)}$ [m/s] the velocity, $a^{(i)}$ [m/s²] acceleration, $u^{(i)}$ [m/s²] control input or desired acceleration, and τ_i [s] the mechanical actuation lag.

We write the state vector of a single vehicle- i as $x^{(i)} = [p^{(i)}, v^{(i)}, a^{(i)}]^\top$, which gives the standard state space form

$$\dot{x}^{(i)} = A_c^{(i)}x^{(i)} + B_c^{(i)}u^{(i)}, \quad (1)$$

where $A_c^{(i)}$ and $B_c^{(i)}$ are the dynamics and control input matrices with the mechanical lag term for vehicle- i and are given in Appendix VIII-A.

Following [72], a continuous-time system (1), can be discretized with sampling interval Δ_t [s] to

$$x_{k+1}^{(i)} = A^{(i)}x_k^{(i)} + B^{(i)}u_k^{(i)} + w_k^{(i)}$$

where the subscript k is discrete-time, $w_k^{(i)}$ is i.i.d. process noise representing error in the discrete-time prediction model, modeled as zero mean normally distributed with covariance $\mathcal{W}^{(i)} > 0$, $w_k^{(i)} \sim \mathcal{N}(0, \mathcal{W}^{(i)})$ and the dynamics matrices are discretized using

$$A^{(i)} = \exp(A_c^{(i)}\Delta_t) \text{ and } B^{(i)} = \int_0^{\Delta_t} \exp(A_c^{(i)}m)dm B_c^{(i)}$$

and are given in Appendix VIII-A.

Following [4], [67] and [16], we consider a MIMO model of the platoon. For M vehicles, we define the centralized multiple-output state and multiple-input control vectors as

$$X_k = [p_k^{(1)}, \dots, p_k^{(M)}, v_k^{(1)}, \dots, v_k^{(M)}, a_k^{(1)}, \dots, a_k^{(M)}]^\top \quad (2)$$

$$U_k = [u_k^{(1)}, \dots, u_k^{(M)}]^\top \quad (3)$$

such that the MIMO platoon dynamics are

$$X_{k+1} = A_M X_k + B_M U_k + W_k \quad (4)$$

where A_M and B_M are block diagonal matrices of the collection of single-vehicle dynamics and control input matrices, W_k is a vector of the i.i.d. process noise acting on each vehicle which can be modeled as $W_k \sim \mathcal{N}(0, \mathcal{W})$. The matrices A_M and B_M are given in Appendix VIII-B.

In MPC design, instead of directly computing the platoon control action U_k , we optimize for the change in control

actions ΔU_k from the previous control action such that the applied control (3) to the platoon dynamics (4) is

$$U_k = U_{k-1} + \Delta U_k \quad (5)$$

where $\Delta U_k = [\Delta u_k^{(1)}, \dots, \Delta u_k^{(M)}]^\top$ and $\Delta u_k^{(i)}$ is the change in control action for vehicle- i . This optimization is computed over a finite horizon of N time steps into the future. We use the platoon model (4) with (5) to predict the value of the state over the next N time steps. We introduce the predicted state value of the platoon at time $k+j$ for $j \in \{1, \dots, N\}$ from the measured state value at time k using the model denoted as $\hat{X}_{k+j|k}$, with the prediction window defined as

$$\mathcal{X}_k = [\hat{X}_{k+1|k}^\top, \dots, \hat{X}_{k+N|k}^\top]^\top$$

for the predicted value of the change in control from the platoon controller as

$$\Delta \hat{U}_k = [\Delta \hat{U}_{k|k}^\top, \dots, \Delta \hat{U}_{k+N-1|k}^\top]^\top$$

where from the measurement at time k the predicted applied control at time k is $\hat{U}_{k|k} = U_{k-1} + \Delta \hat{U}_{k|k}$ and the predicted control at time $k+j$ is $\hat{U}_{k+j|k} = \hat{U}_{k+j-1|k} + \Delta \hat{U}_{k+j|k}$ for $j = \{1, \dots, N-1\}$.

We note that ΔU_k is the change in control applied at time k , while $\Delta \hat{U}_k$ is the predicted change in control over the finite horizon of length N . The actual applied control action is not necessarily equal to the prediction.

Using algebraic manipulation as illustrated in MPC texts (e.g. [60]) the state prediction of the platoon \mathcal{X}_k can be written as a linear combination of the current state X_k , the previous applied control U_{k-1} and the predicted change in control $\Delta \hat{U}_k$

$$\mathcal{X}_k = \Phi X_k + \lambda U_{k-1} + \Gamma \Delta \hat{U}_k \quad (6)$$

where Φ is the propagation of the state through the dynamics matrix A_M , λ and Γ are the propagation of the control inputs through the dynamics and control matrices, and are given in Appendix VIII-B.

We utilize this state prediction model to design a centralized MPC for the coordinated control of a platoon of vehicles.

B. Human Driver Model

We consider during operation of the platoon that a human driver temporarily takes control of their vehicle and desire our platoon controller to reconfigure to this human driver. We assume that the human driver is solely focused on the state of their own vehicle, does not interact with any other vehicles in the platoon, and issues control actions that are consistent with physical (engine limit) and legal (road speed limit) constraints.

Consider a vehicle- ℓ has temporarily left the platoon and has the change in control action $\Delta u_k^{(\ell)}$ from the human driver replacing the platoon control $\Delta u_k^{(\ell)}$ such that the control action is $u_k^{(\ell)} = u_{k-1}^{(\ell)} + \Delta u_k^{(\ell)}$. For ease of notation we modify the platoon change in control action (5) with a switch

$$U_k = U_{k-1} + \alpha_k \Delta U_k + \bar{\alpha}_k \Delta \mathcal{U}_k \quad (7)$$

where $\Delta \mathcal{U}_k$ is the control action applied from a human driver, a binary switch α_k as a diagonal square matrix of size M

that takes ones on the diagonal for the vehicles controlled by the platoon and zero in the i, i th element when vehicle- i is not controlled by the platoon controller, and $\bar{\alpha}_k = I_M - \alpha_k$. When the platoon is fully controlled by the centralized platoon controller $\alpha_k \equiv I_M$ and $\bar{\alpha}_k \equiv 0_M$, and (7) reduces to (5). The dynamics of the platoon (4) are now

$$X_{k+1} = A_M X_k + B_M U_{k-1} + B_M \alpha_k \Delta U_k + B_M \bar{\alpha}_k \Delta \mathcal{U}_k. \quad (8)$$

Based on the applied control at time $k-1$, the platoon controller is aware if every vehicle has utilized the centralized platoon controller or an alternative control value. As such, α_k is known to the controller at time k . If a vehicle has temporarily left the platoon, we assume that the vehicle will continue to be human controlled until informed otherwise, and α_k is constant for the finite prediction horizon. The finite horizon prediction for the state of the platoon is expanded from (6) to

$$\mathcal{X}_k = \Phi X_k + \lambda U_{k-1} + \Gamma (I_N \otimes \alpha_k) \Delta \hat{U}_k + \Gamma (I_N \otimes \bar{\alpha}_k) \Delta \hat{\mathcal{U}}_k \quad (9)$$

where $\Delta \hat{\mathcal{U}}_k = [\Delta \hat{\mathcal{U}}_{k|k}^\top, \dots, \Delta \hat{\mathcal{U}}_{k+N-1|k}^\top]^\top$ are future change in controls from the human driver, \otimes is the Kronecker product and I_N is the identity matrix of size N .

In the following, we design a finite horizon cost function to find the optimal change in control ΔU_k for the platoon, which requires knowledge of any human driver control action $\Delta \mathcal{U}_k$. Ideally for the platoon controller, the future human driver change in control actions $\Delta \mathcal{U}_k$ are known exactly, however, this is unlikely to be the case. This motivates the use of a predicted control action for the human driver control values. For a finite prediction horizon of length N , the human-driver model can be written as

$$\mathcal{X}_k^{(\ell)} = \bar{\Phi} x_k^{(\ell)} + \bar{\lambda} u_{k-1}^{(\ell)} + \bar{\Gamma} \Delta \hat{\mathcal{U}}_k^{(\ell)}, \quad (10)$$

where $\bar{\Phi}$, $\bar{\lambda}$ and $\bar{\Gamma}$ are given in Appendix VIII-B and $\Delta \hat{\mathcal{U}}_k^{(\ell)} = [\Delta \hat{u}_{k+1|k}^{(\ell)}, \dots, \Delta \hat{u}_{k+N-1|k}^{(\ell)}]^\top$ is the prediction of the human driver change in control of vehicle- ℓ .

We utilize (10) to compute a basic prediction of the human driver's control action, which we can utilize in the platoon prediction model (9) to design a centralized MPC for the coordinated control of a platoon of vehicles

IV. PLATOONING PROBLEM

We desire to control the entire platoon to reach a target velocity of v_d [m/s] with the desired distance between vehicle- i and its immediate predecessor vehicle- $(i-1)$ as

$$\bar{d}_k^{(i)} \triangleq d_i + h_k^{(i)} v_k^{(i)} = l_{i-1} + r_i + h_k^{(i)} v_k^{(i)} \quad (11)$$

where $d_i = l_{i-1} + r_i$ is the constant inter-vehicle distance, l_{i-1} [m] is the length of vehicle- $(i-1)$, r_i [m] the desired standstill distance in front of vehicle- i , and $h_k^{(i)}$ [s] the desired headway time. The desired distance r_i and headway time $h_k^{(i)}$ are vehicle specific and input by the respective driver, whereas the desired velocity is platoon specified.

We consider a unique headway time for each vehicle, which can be modified by the occupants of the vehicle. Commercially available ACC systems allow for user selection of headway

time [8], with increments at 1, 1.5 and 2 seconds [73]. We write the individual headways, $h_k^{(i)}$, as a function of time k , to indicate that these can be modified but consider that a reasonable driver would not be constantly changing their headway.

Additionally, we desire to ensure the following constraints for all vehicles $i \in \{1, M\}$

- $p^{(i-1)} - p^{(i)} \geq d_{\min}$, minimum safe distance between vehicles to ensure that no vehicle impacts its predecessor,
- $p^{(i-1)} - p^{(i)} \leq d_{\max}$ maximum distance between vehicles to ensure (random) communications are maintained,
- $v_{\min} \leq v^{(i)}$, minimum velocity set to zero on the assumption that no vehicle in the platoon reverses on the road,
- $v^{(i)} \leq v_{\max}$, maximum velocity chosen based on the road speed limit, or the performance limitation of a vehicle,
- $a_{\min} \leq a^{(i)}$, minimum acceleration bounded based on the performance of the braking systems, and
- $a^{(i)} \leq a_{\max}$, maximum acceleration chosen based on the engine performance of the vehicles.

The acceleration bounds could be further limited for the comfort of the vehicle occupants.

Finally, we also consider that our proposed controller can accommodate a human driver taking temporary control of their vehicle within the platoon. This could include a driver initiating an emergency brake, reducing speed, or temporarily maintaining a larger distance from the previous vehicle than specified. This accommodation allows for a human driver to drive within the bounds of the platoon to their own comfort. Additionally, it may allow for the inclusion of legacy vehicles. We make the minimum assumption that the vehicle and driver will obey performance limits of the vehicle: the minimum and maximum accelerations, and legal limits on velocity: non-negative and not exceeding the road speed limit.

We consider the situation where it is more important for the platoon to stay together, and an emergency brake for one vehicle should be obeyed by the platoon. This is in contrast to control policies in [5], [58] where each vehicle has individual goals and platoons are allowed to split and reform. We desire to ensure that our proposed controller yields a stable closed-loop to temporary inputs from a human driver to their individual vehicle within the minimum constraints.

V. CONTROLLER DESIGN

We now design our controller using the models given above to achieve the desired platoon velocity and inter-vehicle distances while guaranteeing the constraints and able to reconfigure to a temporary human driver. First, we design a time-varying reference for the platoon. Second, we design our finite horizon cost function for the platoon inspired by the infinite horizon cost function of [16]. We also apply the desired constraints on the cost function to propose our constrained MPC controller to centrally control the platoon to the desired platoon velocity and inter-vehicle distances. Third, we propose a simple finite horizon cost function to predict the human driver control actions for use in the platoon controller.

A. Reference Design

Proportional state feedback controllers have been used in several platooning works [39], including LQR [4], [16], [58], [67], [70]. However, for constant gain feedback regulators, the control value increases the further the states are from the desired reference [74]. In reasonable platooning scenarios [69], such as zero initial velocity, the desired initial control actions could exceed maximum allowable control action [75].

To avoid this issue with constant static references used in regulators, we propose a time-varying reference for the desired platoon states. Using a slowly increasing reference, all vehicles in the platoon are able to converge to the desired reference before the position and velocity references reach the desired steady-state. This allows convergence to the reference from any initial condition.

We consider a slowly increasing ramp for the velocity reference with constant acceleration from initial time k_0 as

$$v_k^* = \begin{cases} a_k^* \Delta_t k + \bar{v}, & k_0 \leq k < k_0 + k_m \\ v_d, & k \geq k_0 + k_m \end{cases}$$

where $\bar{v} = \min v_{k_0}^{(i)}$ is initialized to the minimum velocity of the platoon, the acceleration reference is

$$a_k^* = \begin{cases} \frac{v_d - \bar{v}}{\Delta_t k_m}, & k_0 \leq k < k_0 + k_m \\ 0, & k \geq k_0 + k_m \end{cases}$$

and k_m is the sampling periods to reach the desired velocity. The time constant k_m is a tuning parameter of the controller.

For the position reference, we take inspiration from [67] and [16] to establish the position reference of all the vehicles as the cumulative sum of the desired distances from a virtual lead vehicle-0. The lead vehicle position reference is

$$p_k^* = \begin{cases} \frac{1}{2} a_k^* (\Delta_t k)^2 + \bar{v} \Delta_t k + \bar{p}, & k_0 \leq k < k_0 + k_m \\ v_d \Delta_t k + \bar{p}, & k \geq k_0 + k_m \end{cases}$$

where $\bar{p} = p_{k_0}^{(1)} + \bar{d}_{k_0}^{(1)}$ is initialized from the position of vehicle-1. The position reference for each vehicle- i is

$$p_k^{(i)*} = p_k^* - \sum_{j=1}^i \bar{d}_k^{(j)} = p_k^* - \left(\sum_{j=1}^i d_j + h_k^{(j)} v_k^* \right),$$

where $\bar{d}_k^{(i)}$ is defined in (11). By using the desired inter-vehicle distance to form the position reference for each individual vehicle, the headway times are included as part of the state reference.

When a vehicle leaves the platoon under human driver control, we desire to drive the platoon forward at the desired velocity but within the platoon constraints. We reset the platoon reference based on the human controlled vehicle state. The initial time is set as $k_0 = k$, and velocity reference is set to the velocity of vehicle- ℓ : $\bar{v} = v_{k_0}^{(\ell)}$, and the virtual lead vehicle position as the desired distance from vehicle- ℓ : $\bar{p} = p_k^{(\ell)} + \sum_{j=1}^{\ell} \bar{d}_k^{(j)}$.

For convenience we define our desired reference for the platoon at time k as the vector $X_k^* = [p_k^{(1)*}, \dots, p_k^{(M)*}, v_k^*, \dots, v_k^*, a_k^*, \dots, a_k^*]^T$ and over the finite prediction horizon as $\mathcal{X}_k^* = [(X_{k+1}^*)^T, \dots, (X_{k+N}^*)^T]^T$.

B. Cost Function Design

To design our MPC platoon controller we establish position, velocity and acceleration error states using our time-varying references. We propose a finite horizon cost function of these errors and discuss how our cost function can be rearranged to be in a quadratic function of the vehicle states. Finally, we apply the desired inter-vehicle distance, velocity, and acceleration limits as state constraints on the cost function. Our final constrained cost function is in the form of a quadratic program, which can then be solved using standard convex optimization techniques. The constraints on the states and control are a boundary in the cost function solution space, such that the predicted optimal control action is guaranteed to not exceed the desired constraints. There exist several quadratic programming solvers to establish the optimal control action within constraints [60] which reduces to optimization of a convex function [76].

Consider for each vehicle- i for $i \in \{1, M\}$, the absolute position, velocity, and acceleration errors as the difference between the current state and desired reference $\xi_k^{(i)} = p_k^{(i)} - p_k^{(i)*}$, $\zeta_k^{(i)} = v_k^{(i)} - v_k^{(i)*}$, and $\psi_k^{(i)} = a_k^{(i)} - a_k^{(i)*}$. For the entire platoon, these errors can be written as $X_k - X_k^* = [\xi_k^{(1)}, \dots, \xi_k^{(M)}, \zeta_k^{(1)}, \dots, \zeta_k^{(M)}, \psi_k^{(1)}, \dots, \psi_k^{(M)}]$. For convenience below, we define $\hat{\eta}_{k+j|k}^{(i)}$, $\hat{\xi}_{k+j|k}^{(i)}$, $\hat{\zeta}_{k+j|k}^{(i)}$, and $\hat{\psi}_{k+j|k}^{(i)}$ as the predicted errors where the subscript indicates the state prediction at time $k+j$ given the state at time k .

Following [67] and [16], we introduce virtual reference vehicles on the platoon boundary that perfectly follow the reference $p_k^{(0)} = p_k^*$, $p_k^{(M+1)} = p_k^{(M+1)*}$, $v_k^{(0)} = v_k^{(M+1)} = v_k^*$, and $a_k^{(0)} = a_k^{(M+1)} = a_k^*$, and introduce the relative position error between vehicle- i and vehicle- $(i-1)$ for $i \in \{1, M+1\}$ as

$$\eta_k^{(i)} = p_k^{(i)} - p_k^{(i-1)} + \bar{d}_k^{(i)}. \quad (12)$$

Inspired by the infinite horizon cost function of [16] we propose a finite horizon cost function over a prediction horizon of N steps with our time-varying references

$$J = \sum_{j=0}^{N-1} \left[\sum_{i=1}^{M+1} q_1 \left(\hat{\eta}_{k+j|k}^{(i)} \right)^2 + \sum_{i=1}^M \left(q_2 \left(\hat{\xi}_{k+j|k}^{(i)} \right)^2 + q_3 \left(\hat{\zeta}_{k+j|k}^{(i)} \right)^2 + q_4 \left(\hat{\psi}_{k+j|k}^{(i)} \right)^2 + r \left(\Delta u_{k+j|k}^{(i)} \right)^2 \right] + (\hat{X}_{k+N|k} - X_{k+N}^*)^\top P_{k+N} (\hat{X}_{k+N|k} - X_{k+N}^*) \quad (13)$$

where q_1 is the penalty on relative position error, q_2 the penalty on absolute position error, q_3 the penalty on velocity error, q_4 as the penalty on the acceleration, r the penalty on the control inputs, and P_{k+N} is the terminal state cost. To achieve convergence independent of platoon length, it is necessary to penalize both the relative and the absolute position errors [16].

Using algebraic manipulation and (11), the relative position error (12), can be written as a function of the errors

$$\eta_k^{(i)} = \xi_k^{(i)} - \xi_k^{(i-1)} + h_k^{(i)} \zeta_k^{(i)}$$

such that the relative position errors can be incorporated as cross-terms of the absolute position errors and velocity errors, with the headway times as a weight on the velocity errors.

While one could think of the headway times as a reference to the problem as introduced in the desired inter-vehicle distance (11), it is more convenient as a weight on the state deviation. By forcing the headway time to be a state reference, it may lead to a nonlinear control problem.

Our cost function can now be efficiently written as a quadratic function

$$J = (\hat{X}_{k+N|k} - X_{k+N}^*)^\top P_{k+N} (\hat{X}_{k+N|k} - X_{k+N}^*) + \sum_{j=0}^{N-1} \left((\tilde{X}_{k+j|k})^\top Q_{k+j} \tilde{X}_{k+j|k} + \Delta \hat{U}_{k+j|k}^\top R_\Delta \Delta \hat{U}_{k+j|k} \right) \quad (14)$$

where $\tilde{X}_{k+j|k} = \hat{X}_{k+j|k} - X_{k+j}^*$, $R_\Delta = rI_M$, and

$$Q_\kappa = \begin{bmatrix} q_1 T_M + q_2 I_M & q_1 T_\kappa & 0 \\ q_1 T_\kappa^\top & q_1 H_\kappa + q_3 I_M & 0 \\ 0 & 0 & q_4 I_M \end{bmatrix} \quad (15)$$

where 0 is a square matrix of zeros of size $M \times M$, T_M is a symmetric Toeplitz matrix of size $M \times M$ with the first row of the form $[2, -1, 0, \dots, 0]$, T_κ is an $M \times M$ matrix with the headway times of all vehicles $[h_\kappa^{(1)}, \dots, h_\kappa^{(M)}]$ on the diagonal and negative headway times of vehicles-2 to- M on the first upper diagonal $[-h_\kappa^{(2)}, \dots, -h_\kappa^{(M)}]$, and H_κ is a diagonal $M \times M$ matrix where $H_\kappa = \text{diag}[(h_\kappa^{(1)})^2, \dots, (h_\kappa^{(M)})^2]$. In the case of a common constant time headway across the platoon $h_k^{(i)} = h$, then T_κ reduces to a Toeplitz matrix with h on the diagonal and $-h$ on the first upper diagonal, and H_κ reduces to the identity I_M multiplied by h^2 . The reduction of (13) to (14) is given in Appendix VIII-C.

The terminal penalty P is the penalty on the final state in the prediction horizon. Choosing P as the solution of the algebraic Riccati equation implements the infinite horizon cost on the final state such that the final control action is the infinite horizon optimal control action [77].

Using algebraic manipulation and (9) the cost function can be written in the form of a quadratic program

$$J(X_k, \Delta \hat{U}_k) = f(X_k, U_{k-1}) + \Delta U_k^\top (I_N \otimes \alpha_k)^\top (\Psi + \Gamma^\top \Omega \Gamma) (I_N \otimes \alpha_k) \Delta U_k + 2(\Phi X_k + \lambda U_{k-1} + \Gamma(I_N \otimes \bar{\alpha}_k) \Delta \hat{U}_k - \mathcal{X}_k^*)^\top \times \Omega \Gamma (I_N \otimes \alpha_k) \Delta \hat{U}_k \quad (16)$$

where $f(X_k, U_{k-1})$ is a constant term and $\Omega = \text{diag}\{Q_k, \dots, Q_{k+N-1}, P_{k+N}\}$, $\Psi = \text{diag}\{R_\Delta, \dots, R_\Delta\}$ are block diagonal matrices. Thus the optimization problem to be solved using a standard quadratic program solver.

Consider the desired constraints on the vehicles' velocities and accelerations, and the inter-vehicle distances outlined in Section IV of inter-vehicle distance $d_{\min} \leq p_k^{(i-1)} - p_k^{(i)} \leq d_{\max}$, velocity $v_{\min} \leq v_k^{(i)} \leq v_{\max}$, and acceleration $a_{\min} \leq a_k^{(i)} \leq a_{\max}$ which can be written as a matrix inequality of the platoon state

$$\begin{bmatrix} \check{G} & g \\ & \hat{X}_{k+j|k} \\ & & 1 \end{bmatrix} \leq 0$$

where

$$[\check{G} \quad g] = \begin{bmatrix} \check{\mathfrak{T}}_M & 0 & 0 & 1_{M-1}d_{\min} \\ -\check{\mathfrak{T}}_M & 0 & 0 & -1_{M-1}d_{\max} \\ 0 & -I_M & 0 & 1_M v_{\min} \\ 0 & I_M & 0 & -1_M v_{\max} \\ 0 & 0 & -I_M & 1_M a_{\min} \\ 0 & 0 & I_M & -1_M a_{\max} \end{bmatrix}$$

where $\check{\mathfrak{T}}_M$ is a size $(M-1) \times M$ Toeplitz matrix with -1 on the diagonal and 1 on the first upper diagonal, and 1_{M-1} and 1_M are column vectors of ones of size $(M-1)$ and M , respectively, for a total of $6M-2$ constraints for each step of the prediction horizon.

These constraints can be extended over the finite prediction horizon

$$[\bar{G} \quad \bar{g}] \begin{bmatrix} \mathcal{X}_k \\ 1 \end{bmatrix} \leq 0$$

where $\bar{G} = \text{diag}\{\check{G}, \dots, \check{G}\}$ and $\bar{g}^\top = [g^\top, \dots, g^\top]$. Using the prediction model (9), the constraints on the states can be written in terms of $\Delta\hat{U}_k$

$$\begin{aligned} & \bar{G}\Gamma(I_N \otimes \alpha_k)\Delta\hat{U}_k \\ & \leq -\bar{G} \left(\Phi X_k + \lambda U_{k-1} + \Gamma(I_N \otimes \bar{\alpha}_k)\Delta\hat{U}_k \right) - \bar{g} \end{aligned} \quad (17)$$

such that the state constraints appear as a boundary on the cost function (16) [60].

The left of (17) only changes in the event a vehicle leaves the platoon and $\alpha_k \neq \alpha_{k-1}$. For a finite platoon of M vehicles, it is possible to pre-compute all variations of $\bar{G}\Gamma(I_N \otimes \alpha_k)$. However, the right of (17) is dependent on the current platoon state X_k and the last control action U_{k-1} , and must be recomputed each step.

For a prediction horizon of length N , there are $N \times (6M-2)$ constraints across the $3M$ states at each time point, such that \bar{G} is of size $N(6M-2) \times 3MN$. Increasing the length of the prediction horizon, N , increases the number of control values required by the number of vehicles M but the number of applied constraints by $6M-2$. For a large platoon this can be a significant increase to the computational effort.

The optimal platoon control action is the change in control that minimizes the constrained finite horizon cost function

$$\Delta\hat{U}_k^* = \min_{\Delta\hat{U}_k} J(X_k, \Delta\hat{U}_k)$$

which we then implement the first element $\Delta\hat{U}_{k|k}^*$ to the platoon, before solving again at the next time step.

C. Incorporation of Hybridized Human Driver Model

We now predict the change in control action from a human driver for use in our platoon controller using a second MPC algorithm. We assume that the human driver will behave reasonably by rarely changing their control action, and will at minimum obey performance (acceleration) constraints of the vehicle, maintain non-negative velocity and obey the road speed limit. The same acceleration and velocity assumptions are applied in our centralized platoon controller. Future change in control actions from the human driver that violate these constraints could cause our platoon controller to be infeasible.

Using our assumption that the human driver will rarely change their control action, we consider that the driver's control actions will be constant over the finite horizon prediction time of the platoon controller. We propose the following quadratic finite horizon cost function of the human driver

$$\begin{aligned} \bar{J}(x_k^{(\ell)}, \Delta\hat{U}_k^{(\ell)}) &= (\hat{x}_{k+N|k}^{(\ell)})^\top \bar{P}(\hat{x}_{k+N|k}^{(\ell)}) \\ &+ \sum_{j=0}^{N-1} \left((\hat{x}_{k+j|k}^{(\ell)})^\top \bar{Q}(\hat{x}_{k+j|k}^{(\ell)}) + (\Delta\hat{u}_{k+j|k}^{(\ell)})^\top r_\Delta(\Delta\hat{u}_{k+j|k}^{(\ell)}) \right) \end{aligned}$$

As we make no assumption on desired state of the human driver, we choose no penalty on the state such that $\bar{Q} = \bar{P} = 0$, and the penalty on control action as the same in the platoon model, where r_Δ is the ℓ th diagonal element of R_Δ .

This is the most uninformative cost function possible as it assumes the driver will make no changes to their current control value. Our cost function simplifies to

$$\bar{J}(x_k^{(\ell)}, \Delta\hat{U}_k^{(\ell)}) = (\Delta\hat{U}_k^{(\ell)})^\top \bar{\Psi} \Delta\hat{U}_k^{(\ell)} \quad (18)$$

where $\bar{\Psi} = \text{diag}\{r_\Delta, \dots, r_\Delta\}$. Clearly, this cost function is minimized when $\Delta\hat{U}_k^{(\ell)} = 0$.

Now we consider the minimum constraints that we assume the human driver obeys of velocity $v_{\min} \leq v_k^{(\ell)} \leq v_{\max}$, and acceleration $a_{\min} \leq a_k^{(\ell)} \leq a_{\max}$ which can be written as the matrix inequality on the vehicle state at time $k+j$ as

$$[\check{G} \quad g] \begin{bmatrix} \hat{x}_{k+j|k}^{(\ell)} \\ 1 \end{bmatrix} \leq 0$$

where

$$[\check{G} \quad g] = \begin{bmatrix} 0 & -1 & 0 & v_{\min} \\ 0 & 1 & 0 & -v_{\max} \\ 0 & 0 & -1 & a_{\min} \\ 0 & 0 & 1 & -a_{\max} \end{bmatrix}.$$

These constraints can be extended over the finite prediction horizon

$$[\bar{G} \quad \bar{g}] \begin{bmatrix} \mathcal{X}_k^{(\ell)} \\ 1 \end{bmatrix} \leq 0$$

where $\bar{G} = \text{diag}\{\check{G}, \dots, \check{G}\}$ and $\bar{g}^\top = [g^\top, \dots, g^\top]$. Using algebraic manipulation with the vehicle dynamics (10), the state constraints can be written as a function of the change in control $\Delta\hat{U}_k^{(\ell)}$

$$\bar{G}\Gamma\Delta\hat{U}_k^{(\ell)} \leq -\bar{G} \left(\bar{\Phi}x_k^{(\ell)} + \bar{\lambda}u_{k-1}^{(\ell)} \right) - \bar{g}. \quad (19)$$

The quadratic cost function (18) with the linear matrix constraints (19) can be minimized using standard quadratic programming solvers to find the minimum control action that meets the constraints. We would only expect to predict a change in control action when one of the constraints will be violated in the finite horizon. We take the prediction of the constrained but minimally penalized control for the human controlled vehicle- ℓ , $\Delta\hat{U}_k^{(\ell)}$, and include this in the computation for the centralized platoon control action.

VI. NUMERICAL STUDY

In this section, we provide guidance on the tuning of the cost function weights from (13) and illustrate a numerical experiment.

A. Cost Function Weights

Increasing all of the penalties, q_1 , q_2 , q_3 , and q_4 , substantially can cause the optimization algorithm to become infeasible. In the initial transient phase, it is necessary for the vehicles to deviate from the desired reference to enable convergence of both positions and velocities. However, it is the initial transient phase where the impact is most prominent.

At minimum the position error penalties q_1 and q_2 are required to ensure that the platoon converges to the desired positions. As motivated in [16], the absolute position error penalty q_2 must be present or convergence is a function of the platoon length. As q_1 and q_2 affect the same position error states, it is suggested to tune these parameters together. Increasing q_2 forces the vehicles to the position reference, with less regard to the relative distance, while increasing the relative position error penalty q_1 preferences the inter-vehicle distance over the absolute position reference.

It is possible to set the velocity (q_3) and acceleration (q_4) penalties to zero. Convergence is natural following the position errors. Increasing the velocity (acceleration) penalty forces the velocities (accelerations) closer to the desired reference, which can slow the convergence of the positions.

B. Numerical Simulation

We consider a numerical simulation of five ($M = 5$) vehicles, with sampling period of $\Delta_t = 0.5$ [sec/sample]. We consider vehicle lengths as $l_i = 2.5$ [m] for all $i = \{1, 5\}$, and the vehicle mechanical lags as $\tau_1 = 0.5$, $\tau_2 = 0.2$, $\tau_3 = 0.3$, $\tau_4 = 0.6$, and $\tau_5 = 0.4$ [sec]. The desired velocity is set as highway speed limit of 100 [km/h] or $v_d = 27.78$ [m/s]. We consider that the drivers individually select their desired standstill distances as $r_1 = 6$, $r_2 = 6$, $r_3 = 5$, $r_4 = 8$, $r_5 = 7$ [m], and headway times as $h_k^{(1)} = 1$, $h_k^{(2)} = 1.3$, $h_k^{(3)} = 1.5$, $h_k^{(4)} = 0.8$, and $h_k^{(5)} = 1.2$ [sec].

At time 100 [sec] the human driver of vehicle-3 performs an emergency brake to 0 [m/s], then at 150 [sec] increases speed to 11 [m/s] until 250 [sec] when the driver returns to platoon control. Following this disturbance at 320 [sec] the drivers in the platoon increase their inter-vehicle distance for additional safety and set their headway times to $h_k^{(2)} = 3$, $h_k^{(3)} = 2.6$, $h_k^{(4)} = 4$, and $h_k^{(5)} = 2.5$ [sec].

We choose the acceleration constraints as $a_{\min} = -6$ [m/s²] and $a_{\max} = 3$ [m/s²], based on the performance of an average passenger vehicle and comfort of passengers. We choose the velocity constraints as $v_{\min} = 0$ [m/s] and $v_{\max} = 27.8$ [m/s], based on the road speed limit. Finally, the minimum inter-vehicle distance $d_{\min} = 2$ [m] and maximum inter-vehicle distance $d_{\max} = 130$ [m]. We consider a prediction horizon of 10 [sec], or 20 samples, with time to reach desired velocity of 20 [sec] which equates to $k_m = 40$ [samples].

We choose the penalty on the relative position errors or inter-vehicle distances as $q_1 = 1$, absolute position error or error to the position reference as $q_2 = 1$, velocity errors as $q_3 = 1$, acceleration errors as $q_4 = 1$, change in control as $R_\Delta = 2I_M$, and the terminal cost P as the solution to the algebraic Riccati equation.

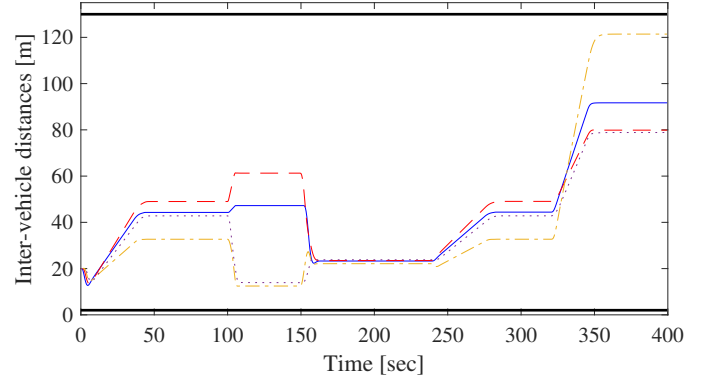


Figure 3. Inter-vehicle Distance from simulation with five vehicles. The inter-vehicle distance smoothly converges to the desired values. The distances between each pair of neighboring vehicles are represented by colors (solid blue, dashed red, dash-dot yellow, dotted purple).

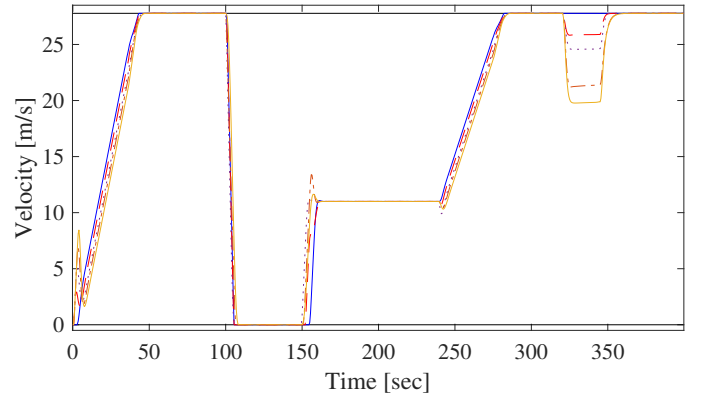


Figure 4. Velocities of the five vehicles from the simulation. The vehicles follow the velocity reference and are represented by colors (solid blue, dashed red, dotted purple, dash-dot brown, solid gold).

We simulate our proposed control design using a standard convex optimizer¹ to solve the quadratic program (16) with constraints (17), and human driver prediction (18) with (19). The inter-vehicle distances are shown in Figure 3, with constraints as the solid horizontal black lines. Our proposed controller converges the vehicles to the desired inter-vehicle distances by converging to the desired position reference and ensures the position constraints are maintained when vehicle-3 is controlled by a human driver.

Figure 4 shows the velocities and Figure 5 shows the accelerations of the five vehicles in the platoon. Our proposed controller smoothly converges the vehicles to the target velocity reference. We observe that the controller initially accelerates the latter vehicles in the platoon to converge to the target position references.

Our simulation results illustrate that our proposed control design successfully converges the controlled vehicles to the desired velocity and inter-vehicle distances. The controller smoothly accelerates the vehicles to the desired position reference, minimizing the absolute and relative distance errors before converging the velocity to the reference. The use of a constrained MPC optimization approach ensures that safety

¹We use the MATLAB `mpcActiveSetSolver` from the MPC toolbox.

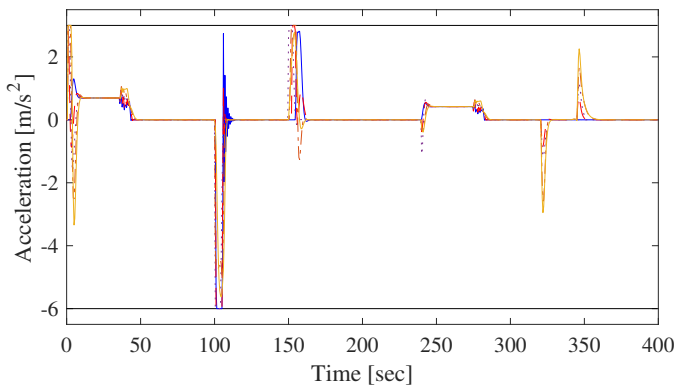


Figure 5. Accelerations of the five vehicles from the simulation. The vehicles follow the acceleration reference and are represented by colors (solid blue, dashed red, dotted purple, dash-dot brown, solid gold).

margins on the inter-vehicle are maintained, the velocities are within the road speed limits, and the commanded accelerations are appropriate for the vehicle and comfortable for passengers.

Our simulation also illustrates that the use of a human driver model enables the centralized controller to operate in the presence of an unknown human driver. Within one sample, our centralized controller reacts, reducing the speed of the remaining vehicles and ensuring that all vehicles in the platoon reach zero velocity before the inter-vehicle distance constraints are violated. As the human controlled vehicle speeds up to a slower velocity than desired, the platoon then maintains the desired standstill distances with additional margin of the constant time headway while matching the lower velocity. As the vehicle returns the platoon smoothly accelerates back to the desired velocity and returns to the full desired inter-vehicle distances.

Remark: Without the inclusion of the human driver in the model (9), a simpler platoon controller would be unable to react to the human driver actions. A state feedback control using (4), would command vehicles-4 and-5 to drive through vehicle-3, and give a constraint violating instruction to vehicle-3 when it re-joined the platoon. An MPC designed for the platoon using (6), would become infeasible as the actions of vehicle-3 would result in its state being constraint violating.

VII. CONCLUSION

In this paper we propose a hybrid constrained MPC algorithm to control a heterogeneous platoon of vehicles to a desired platoon velocity and inter-vehicle distance. The finite horizon cost function of our centralized platoon controller is inspired from the the infinite horizon cost function of [16] with inclusion of headway times individual to each vehicle and able to be changed with time. In our approach, we propose the use of a cost function to predict the control actions of a human driver that takes control of their vehicle, by assuming that the human driver will only obey at minimum, the legal velocity limits and the physical performance constraints of their vehicle. We illustrate the performance of our control approach in a numerical study. Future work includes implementing state estimation such that a centralized approach can be operated

decentralized, with consideration of unreliable communication between vehicles.

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VIII. APPENDIX

A. Single Vehicle Dynamics

The continuous-time dynamics and control input matrices of the dynamics (1) are

$$A_c^{(i)} \triangleq \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau_i} \end{bmatrix}, \quad \text{and} \quad B_c^{(i)} \triangleq \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\tau_i} \end{bmatrix}.$$

Following [72], a continuous-time system can be discretized with sampling interval Δ_t [s] using

$$A^{(i)} = \exp(A_c^{(i)} \Delta_t) \quad \text{and} \quad B^{(i)} = \int_0^{\Delta_t} \exp(A_c^{(i)} m) dm B_c^{(i)}$$

to give

$$A^{(i)} = \begin{bmatrix} 1 & \Delta_t & \tau_i \left(\Delta_t - \tau_i \left(1 - \exp\left(-\frac{\Delta_t}{\tau_i}\right) \right) \right) \\ 0 & 1 & \tau_i \left(1 - \exp\left(-\frac{\Delta_t}{\tau_i}\right) \right) \\ 0 & 0 & \exp\left(-\frac{\Delta_t}{\tau_i}\right) \end{bmatrix}$$

and

$$B^{(i)} = \begin{bmatrix} -\tau_i \left(\Delta_t - \tau_i \left(1 - \exp\left(-\frac{\Delta_t}{\tau_i}\right) \right) \right) + \frac{\Delta_t^2}{2} \\ \Delta_t - \tau_i \left(1 - \exp\left(-\frac{\Delta_t}{\tau_i}\right) \right) \\ 1 - \exp\left(-\frac{\Delta_t}{\tau_i}\right) \end{bmatrix}$$

B. Platoon Dynamics

The block diagonal dynamics matrices of (4) are defined as

$$A_M = \begin{bmatrix} I_M & \Delta_t I_M & A_M^{(1,3)} \\ 0 & I_M & A_M^{(2,3)} \\ 0 & 0 & A_M^{(3,3)} \end{bmatrix} \quad \text{and} \quad B_M = \begin{bmatrix} B_M^{(1,1)} \\ B_M^{(2,1)} \\ B_M^{(3,1)} \end{bmatrix}$$

where I_M the identity matrix of size $M \times M$, 0 is a matrix of zeros of appropriate size, and

$$\begin{aligned} A_M^{(1,3)} &= \text{diag} \left[\tau_1 \left(\Delta_t - \tau_1 \left(1 - \exp\left(-\frac{\Delta_t}{\tau_1}\right) \right) \right), \dots, \right. \\ &\quad \left. \tau_M \left(\Delta_t - \tau_M \left(1 - \exp\left(-\frac{\Delta_t}{\tau_M}\right) \right) \right) \right] \\ A_M^{(2,3)} &= \text{diag} \left[\tau_1 \left(1 - \exp\left(-\frac{\Delta_t}{\tau_1}\right) \right), \dots, \right. \\ &\quad \left. \tau_M \left(1 - \exp\left(-\frac{\Delta_t}{\tau_M}\right) \right) \right] \\ A_M^{(3,3)} &= \text{diag} \left[\exp\left(-\frac{\Delta_t}{\tau_1}\right), \dots, \exp\left(-\frac{\Delta_t}{\tau_M}\right) \right] \\ B_M^{(1,1)} &= \text{diag} \left[-\tau_1 \left(\Delta_t - \tau_1 \left(1 - \exp\left(-\frac{\Delta_t}{\tau_1}\right) \right) \right) + \frac{\Delta_t^2}{2}, \right. \\ &\quad \left. \dots, -\tau_M \left(\Delta_t - \tau_M \left(1 - \exp\left(-\frac{\Delta_t}{\tau_M}\right) \right) \right) + \frac{\Delta_t^2}{2} \right] \\ B_M^{(2,1)} &= \text{diag} \left[\Delta_t - \tau_1 \left(1 - \exp\left(-\frac{\Delta_t}{\tau_1}\right) \right), \dots, \right. \\ &\quad \left. \Delta_t - \tau_M \left(1 - \exp\left(-\frac{\Delta_t}{\tau_M}\right) \right) \right] \\ B_M^{(3,1)} &= \text{diag} \left[1 - \exp\left(-\frac{\Delta_t}{\tau_1}\right), \dots, 1 - \exp\left(-\frac{\Delta_t}{\tau_M}\right) \right] \end{aligned}$$

In the case of a homogeneous platoon, $\tau_i = \tau$, then $A^{(i)} = A$ and $B^{(i)} = B$ and the platoon dynamics can be conveniently computed as

$$A_M = A \otimes I_M \quad \text{and} \quad B_M = B \otimes I_M$$

where \otimes is the Kronecker operator.

The matrices of the prediction model of the platoon dynamics (6) are

$$\begin{aligned} \Phi &= \begin{bmatrix} A_M \\ \vdots \\ A_M^N \end{bmatrix}, \quad \lambda = \begin{bmatrix} A_M^0 B_M \\ \vdots \\ (A_M^{N-1} + \dots + A_M^0) B_M \end{bmatrix}, \\ \text{and} \quad \Gamma &= \begin{bmatrix} B_M & \dots & 0 \\ \vdots & \ddots & \vdots \\ (A_M^{N-1} + \dots + A_M^0) B_M & \dots & B_M \end{bmatrix} \end{aligned}$$

and for the human controlled vehicle (10) are

$$\bar{\Phi} = \begin{bmatrix} A^{(\ell)} \\ \vdots \\ (A^{(\ell)})^N \end{bmatrix}, \quad \bar{\lambda} = \begin{bmatrix} (A^{(\ell)})^0 B^{(\ell)} \\ \vdots \\ ((A^{(\ell)})^{N-1} + \dots + (A^{(\ell)})^0) B^{(\ell)} \end{bmatrix},$$

and

$$\bar{\Gamma} = \begin{bmatrix} B^{(\ell)} & \dots & 0 \\ \vdots & \ddots & \vdots \\ ((A^{(\ell)})^{N-1} + \dots + (A^{(\ell)})^0) B^{(\ell)} & \dots & B^{(\ell)} \end{bmatrix}$$

C. Cost Function Expansion

In the below, we show the expansion of sums in the cost function from (13) to the matrix version (14). Consider (13)

$$\begin{aligned} J &= \sum_{j=0}^{N-1} \left[\sum_{i=1}^{M+1} q_1 \left(\hat{\eta}_{k+j|k}^{(i)} \right)^2 + \sum_{i=1}^M \left(q_2 \left(\hat{\xi}_{k+j|k}^{(i)} \right)^2 + q_3 \left(\hat{\zeta}_{k+j|k}^{(i)} \right)^2 \right. \right. \\ &\quad \left. \left. + q_4 \left(\hat{\psi}_{k+j|k}^{(i)} \right)^2 + r \left(\Delta u_{k+j|k}^{(i)} \right)^2 \right) \right] \\ &\quad + \begin{bmatrix} \hat{\xi}_{k+N|k} \\ \hat{\zeta}_{k+N|k} \\ \hat{\psi}_{k+N|k} \end{bmatrix}^\top P_{k+N} \begin{bmatrix} \hat{\xi}_{k+N|k} \\ \hat{\zeta}_{k+N|k} \\ \hat{\psi}_{k+N|k} \end{bmatrix} \end{aligned}$$

The first sum in (13) is of the relative position errors $\hat{\eta}_\kappa$ noting that we substitute κ in place of $k+j|k$. We start

by showing this in terms of the absolute position errors and velocity errors

$$\begin{aligned}
& \sum_{i=1}^{M+1} q_1 \left(\hat{\eta}_{\kappa}^{(i)} \right)^2 \\
&= q_1 \sum_{i=1}^{M+1} \left(\hat{\xi}_{\kappa}^{(i)} - \hat{\xi}_{\kappa}^{(i-1)} + h_{\kappa}^{(i)} \hat{\zeta}_{\kappa}^{(i)} \right)^2 \\
&= q_1 \sum_{i=1}^{M+1} \left[\left(\hat{\xi}_{\kappa}^{(i)} \right)^2 + \left(\hat{\xi}_{\kappa}^{(i-1)} \right)^2 + \left(h_{\kappa}^{(i)} \right)^2 \left(\hat{\zeta}_{\kappa}^{(i)} \right)^2 \right. \\
&\quad \left. - 2 \hat{\xi}_{\kappa}^{(i)} \hat{\xi}_{\kappa}^{(i-1)} + 2 h_{\kappa}^{(i)} \hat{\xi}_{\kappa}^{(i)} \hat{\zeta}_{\kappa}^{(i)} - 2 h_{\kappa}^{(i)} \hat{\xi}_{\kappa}^{(i-1)} \hat{\zeta}_{\kappa}^{(i)} \right] \\
&= q_1 \left[\left(\hat{\xi}_{\kappa}^{(1)} \right)^2 + \left(\hat{\xi}_{\kappa}^{(0)} \right)^2 + \left(h_{\kappa}^{(1)} \right)^2 \left(\hat{\zeta}_{\kappa}^{(1)} \right)^2 \right. \\
&\quad \left. - 2 \hat{\xi}_{\kappa}^{(1)} \hat{\xi}_{\kappa}^{(0)} + 2 h_{\kappa}^{(1)} \hat{\xi}_{\kappa}^{(1)} \hat{\zeta}_{\kappa}^{(1)} - 2 h_{\kappa}^{(1)} \hat{\xi}_{\kappa}^{(0)} \hat{\zeta}_{\kappa}^{(1)} \right. \\
&\quad \left. + \left(\hat{\xi}_{\kappa}^{(2)} \right)^2 + \left(\hat{\xi}_{\kappa}^{(1)} \right)^2 + \left(h_{\kappa}^{(2)} \right)^2 \left(\hat{\zeta}_{\kappa}^{(2)} \right)^2 \right. \\
&\quad \left. - 2 \hat{\xi}_{\kappa}^{(2)} \hat{\xi}_{\kappa}^{(1)} + 2 h_{\kappa}^{(2)} \hat{\xi}_{\kappa}^{(2)} \hat{\zeta}_{\kappa}^{(2)} - 2 h_{\kappa}^{(2)} \hat{\xi}_{\kappa}^{(1)} \hat{\zeta}_{\kappa}^{(2)} \right. \\
&\quad \left. + \dots + \left(\hat{\xi}_{\kappa}^{(M)} \right)^2 + \left(\hat{\xi}_{\kappa}^{(M-1)} \right)^2 + \left(h_{\kappa}^{(M)} \right)^2 \left(\hat{\zeta}_{\kappa}^{(M)} \right)^2 \right. \\
&\quad \left. - 2 \hat{\xi}_{\kappa}^{(M)} \hat{\xi}_{\kappa}^{(M-1)} + 2 h_{\kappa}^{(M)} \hat{\xi}_{\kappa}^{(M)} \hat{\zeta}_{\kappa}^{(M)} - 2 h_{\kappa}^{(M)} \hat{\xi}_{\kappa}^{(M-1)} \hat{\zeta}_{\kappa}^{(M)} \right. \\
&\quad \left. + \left(\hat{\xi}_{\kappa}^{(M+1)} \right)^2 + \left(\hat{\xi}_{\kappa}^{(M)} \right)^2 + \left(h_{\kappa}^{(M+1)} \right)^2 \left(\hat{\zeta}_{\kappa}^{(M+1)} \right)^2 \right. \\
&\quad \left. - 2 \hat{\xi}_{\kappa}^{(M+1)} \hat{\xi}_{\kappa}^{(M)} + 2 h_{\kappa}^{(M+1)} \hat{\xi}_{\kappa}^{(M+1)} \hat{\zeta}_{\kappa}^{(M+1)} \right. \\
&\quad \left. - 2 h_{\kappa}^{(M+1)} \hat{\xi}_{\kappa}^{(M)} \hat{\zeta}_{\kappa}^{(M+1)} \right]
\end{aligned}$$

Recall that the virtual lead and tail vehicles perfectly follow the reference such that

$$\begin{aligned}
\hat{\xi}_{\kappa}^{(0)} &= \hat{\xi}_{\kappa}^{(M+1)} = 0 \\
\hat{\zeta}_{\kappa}^{(0)} &= \hat{\zeta}_{\kappa}^{(M+1)} = 0
\end{aligned}$$

then

$$\begin{aligned}
& q_1 \left[\left(\hat{\xi}_{\kappa}^{(1)} \right)^2 + \left(h_{\kappa}^{(1)} \right)^2 \left(\hat{\zeta}_{\kappa}^{(1)} \right)^2 + 2 h_{\kappa}^{(1)} \hat{\xi}_{\kappa}^{(1)} \hat{\zeta}_{\kappa}^{(1)} \right. \\
&\quad \left. + \left(\hat{\xi}_{\kappa}^{(2)} \right)^2 + \left(h_{\kappa}^{(2)} \right)^2 \left(\hat{\zeta}_{\kappa}^{(2)} \right)^2 \right. \\
&\quad \left. - 2 \hat{\xi}_{\kappa}^{(2)} \hat{\xi}_{\kappa}^{(1)} + 2 h_{\kappa}^{(2)} \hat{\xi}_{\kappa}^{(2)} \hat{\zeta}_{\kappa}^{(2)} - 2 h_{\kappa}^{(2)} \hat{\xi}_{\kappa}^{(1)} \hat{\zeta}_{\kappa}^{(2)} \right. \\
&\quad \left. + \dots + \left(\hat{\xi}_{\kappa}^{(M)} \right)^2 + \left(h_{\kappa}^{(M)} \right)^2 \left(\hat{\zeta}_{\kappa}^{(M)} \right)^2 \right. \\
&\quad \left. - 2 \hat{\xi}_{\kappa}^{(M)} \hat{\xi}_{\kappa}^{(M-1)} + 2 h_{\kappa}^{(M)} \hat{\xi}_{\kappa}^{(M)} \hat{\zeta}_{\kappa}^{(M)} - 2 h_{\kappa}^{(M)} \hat{\xi}_{\kappa}^{(M-1)} \hat{\zeta}_{\kappa}^{(M)} \right. \\
&\quad \left. + \left(\hat{\xi}_{\kappa}^{(M+1)} \right)^2 \right] \\
&= q_1 \left[2 \left(\hat{\xi}_{\kappa}^{(1)} \right)^2 + \dots + \left(\hat{\xi}_{\kappa}^{(M)} \right)^2 \right. \\
&\quad \left. - 2 \hat{\xi}_{\kappa}^{(2)} \hat{\xi}_{\kappa}^{(1)} - \dots - 2 \hat{\xi}_{\kappa}^{(M)} \hat{\xi}_{\kappa}^{(M-1)} \right. \\
&\quad \left. + \left(h_{\kappa}^{(1)} \right)^2 \left(\hat{\zeta}_{\kappa}^{(1)} \right)^2 + \dots + \left(h_{\kappa}^{(M)} \right)^2 \left(\hat{\zeta}_{\kappa}^{(M)} \right)^2 \right. \\
&\quad \left. + 2 h_{\kappa}^{(1)} \hat{\xi}_{\kappa}^{(1)} \hat{\zeta}_{\kappa}^{(1)} + \dots + 2 h_{\kappa}^{(M)} \hat{\xi}_{\kappa}^{(M)} \hat{\zeta}_{\kappa}^{(M)} \right. \\
&\quad \left. - 2 h_{\kappa}^{(2)} \hat{\xi}_{\kappa}^{(1)} \hat{\zeta}_{\kappa}^{(2)} - \dots - 2 h_{\kappa}^{(M)} \hat{\xi}_{\kappa}^{(M-1)} \hat{\zeta}_{\kappa}^{(M)} \right]
\end{aligned}$$

This can be written in matrix notation as

$$\begin{aligned}
& q_1 \left(\hat{\xi}_{\kappa}^{\top} T_M \hat{\xi}_{\kappa} + \hat{\zeta}_{\kappa}^{\top} H_{\kappa} \hat{\zeta}_{\kappa} + \hat{\xi}_{\kappa}^{\top} T_{\kappa} \hat{\zeta}_{\kappa} + \hat{\zeta}_{\kappa}^{\top} T_{\kappa}^{\top} \hat{\xi}_{\kappa} \right) \\
&= q_1 \begin{bmatrix} \hat{\xi}_{\kappa} \\ \hat{\zeta}_{\kappa} \end{bmatrix}^{\top} \begin{bmatrix} T_M & T_{\kappa} \\ T_{\kappa}^{\top} & H_{\kappa} \end{bmatrix} \begin{bmatrix} \hat{\xi}_{\kappa} \\ \hat{\zeta}_{\kappa} \end{bmatrix}
\end{aligned}$$

where T_M is a symmetric Toeplitz matrix of size $M \times M$ with the first row of the form $[2, -1, 0, \dots, 0]$, and T_{κ} and H_{κ} are $M \times M$ matrices where

$$T_{\kappa} = \begin{bmatrix} h_{\kappa}^{(1)} & -h_{\kappa}^{(2)} & 0 & \dots & 0 & 0 \\ 0 & h_{\kappa}^{(2)} & -h_{\kappa}^{(3)} & \dots & 0 & 0 \\ \vdots & & \ddots & \ddots & & \vdots \\ \vdots & & & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & h_{\kappa}^{(M-1)} & -h_{\kappa}^{(M)} \\ 0 & 0 & 0 & \dots & 0 & h_{\kappa}^{(M)} \end{bmatrix}$$

and $H_{\kappa} = \begin{bmatrix} \left(h_{\kappa}^{(1)} \right)^2 & \dots & 0 \\ & \ddots & \\ 0 & \dots & \left(h_{\kappa}^{(M)} \right)^2 \end{bmatrix}$.

Returning to the full sum, we can see that the other terms can also be written in matrix notation

$$\begin{aligned}
J &= \sum_{j=0}^{N-1} \left[\sum_{i=1}^{M+1} q_1 \left(\hat{\eta}_{k+j|k}^{(i)} \right)^2 + \sum_{i=1}^M \left(q_2 \left(\hat{\xi}_{k+j|k}^{(i)} \right)^2 \right. \right. \\
&\quad \left. \left. + q_3 \left(\hat{\zeta}_{k+j|k}^{(i)} \right)^2 + q_4 \left(\hat{\psi}_{k+j|k}^{(i)} \right)^2 + r \left(\Delta u_{k+j|k} \right)^2 \right) \right] \\
&\quad + \begin{bmatrix} \hat{\xi}_{k+N|k} \\ \hat{\zeta}_{k+N|k} \\ \hat{\psi}_{k+N|k} \end{bmatrix}^{\top} P_{k+N} \begin{bmatrix} \hat{\xi}_{k+N|k} \\ \hat{\zeta}_{k+N|k} \\ \hat{\psi}_{k+N|k} \end{bmatrix} \\
&= \sum_{j=0}^{N-1} \left[q_1 \begin{bmatrix} \hat{\xi}_{k+j|k} \\ \hat{\zeta}_{k+j|k} \end{bmatrix}^{\top} \begin{bmatrix} T_M & T_{k+j} \\ T_{k+j}^{\top} & H_{k+j} \end{bmatrix} \begin{bmatrix} \hat{\xi}_{k+j|k} \\ \hat{\zeta}_{k+j|k} \end{bmatrix} \right. \\
&\quad \left. + q_2 \hat{\xi}_{k+j|k}^{\top} I_M \hat{\xi}_{k+j|k} + q_3 \hat{\zeta}_{k+j|k}^{\top} I_M \hat{\zeta}_{k+j|k} \right. \\
&\quad \left. + q_4 \hat{\psi}_{k+j|k}^{\top} I_m \hat{\psi}_{k+j|k} + r \left(\Delta U_{k+j|k} \right)^{\top} I_M \Delta U_{k+j|k} \right] \\
&\quad + \begin{bmatrix} \hat{\xi}_{k+N|k} \\ \hat{\zeta}_{k+N|k} \\ \hat{\psi}_{k+N|k} \end{bmatrix}^{\top} P_{k+N} \begin{bmatrix} \hat{\xi}_{k+N|k} \\ \hat{\zeta}_{k+N|k} \\ \hat{\psi}_{k+N|k} \end{bmatrix} \\
&= \sum_{j=0}^{N-1} \left[\begin{bmatrix} \hat{\xi}_{k+j|k} \\ \hat{\zeta}_{k+j|k} \\ \hat{\psi}_{k+j|k} \end{bmatrix}^{\top} Q_{k+j} \begin{bmatrix} \hat{\xi}_{k+j|k} \\ \hat{\zeta}_{k+j|k} \\ \hat{\psi}_{k+j|k} \end{bmatrix} \right. \\
&\quad \left. + \left(\Delta U_{k+j|k} \right)^{\top} R_{\Delta} \Delta U_{k+j|k} \right] \\
&\quad + \begin{bmatrix} \hat{\xi}_{k+N|k} \\ \hat{\zeta}_{k+N|k} \\ \hat{\psi}_{k+N|k} \end{bmatrix}^{\top} P_{k+N} \begin{bmatrix} \hat{\xi}_{k+N|k} \\ \hat{\zeta}_{k+N|k} \\ \hat{\psi}_{k+N|k} \end{bmatrix}
\end{aligned}$$

where from (15) of $R_{\Delta} = r I_M$ and

$$Q_{\kappa} = \begin{bmatrix} q_1 T_M + q_2 I_M & q_1 T_{\kappa} & 0 \\ q_1 T_{\kappa}^{\top} & q_1 H_{\kappa} + q_3 I_M & 0 \\ 0 & 0 & q_4 I_M \end{bmatrix}$$

then with further simplification of state and reference we find the cost function in reduced form (14)

$$\begin{aligned}
 J = & \sum_{j=0}^{N-1} \left((X_{k+j|k} - X_{k+j}^*)^\top Q_{k+j} (X_{k+j|k} - X_{k+j}^*) \right. \\
 & \left. + (\Delta U_{k+j|k})^\top R_\Delta \Delta U_{k+j|k} \right) \\
 & + (X_{k+N|k} - X_{k+N}^*)^\top P_{k+N} (X_{k+N|k} - X_{k+N}^*)
 \end{aligned}$$