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# Multiple Access Interference Mitigation in OFDMA Uplink via static assignment of Guard Bands and Guard Intervals

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Berlin, October 2013

TKN Technical Report TKN-13-004

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TKN Technical Reports Series  
Editor: Prof. Dr.-Ing. Adam Wolisz

## **Abstract**

In the uplink of OFDMA-based systems, Multiple Access Interference (MAI), caused by synchronization offsets, can cause considerable degradation in the performance of the User Terminals (UTs). The conventional approach to deal with MAI is based on the usage of Cyclic Prefix (CP) also known as Guard Intervals (GI) in time domain. The time and frequency offsets of UTs signals arriving at BS are estimated and then these estimations are used to subtract the offsets and to re-build orthogonality among subcarriers. However the large overhead, caused by long CP and pilot subcarriers, and imposed by this approach, is considered as a main drawback of this approach. Another way of mitigating MAI is by inserting frequency Guard Bands (GB) to reduce the MAI on adjacent subcarriers. In this report, we examine the feasibility of using GB together with CP instead of only CP using a simple standard model. We consider a scenario where a fixed-width CP is used with fixed-width GB to mitigate MAI and improve UT throughput. Various combinations of CP and GB lengths are evaluated under different time and frequency offset profiles. It is shown that short CP together with GBs can bring significant improvement in throughput compared to the mere usage of long CP. Particularly, using GBs with short CP achieves the best performance in presence of both frequency and time offset.

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# Chapter 1.

## Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is one of the most popular techniques for wireless communication due to its robustness against fast fading and inter-symbol interference and its spectrum efficiency attained by orthogonality of subcarriers [4]. OFDM structure can be extended to multiple-access scenarios to include multiple user transmission. Orthogonal Frequency Division Multiple Access (OFDMA) is a multiple access technique based on OFDM which has been adopted in different standards such as IEEE 802.16 and Long Term Evolution (LTE). OFDMA is used in both downlink as well as uplink transmissions due to several of its favorable characteristics inherited from OFDM, such as efficient usage of spectrum, robustness against frequency selective fading and flexible resource allocation.

The primary challenge in usage of OFDMA in uplink is achieving synchronization between several User Terminals (UTs) and Base Stations (BS) in both time and frequency domain. The synchronization problem arises from the fact that signals arriving at BS in uplink are superpositions of user signals sent by several UTs simultaneously. Lack of synchronization disturbs the orthogonality and causes Multiple Access Interference (MAI) that can severely degrade performance of the system [5, 9]. In other words, MAI can be considered as the sum of Inter-Symbol Interference (ISI) caused by timing offsets and Inter-Carrier Interference (ICI) caused by frequency offsets.

A common approach to deal with MAI is based on the estimation of time and frequency offsets of UTs signals arriving at BS. These estimations are then used to subtract the offsets and to re-build orthogonality among subcarriers (see [6] and references therein). The estimation consists of two parts. First step is the coarse estimation which derives the offset through a rough estimation at the beginning of each uplink frame and it is normally based on the information embedded in the Cyclic Prefix (CP) [6]. The residual offsets on each OFDMA symbols of uplink frames are then estimated by a fine tracking process, which is build on the basis of pilot subcarriers. To alleviate the synchronization task, CP is recommended to be relatively long in order to deal with delay spread, 2-way-propagation delay and the timing errors caused by the asynchronicity of oscillators [8]. The main disadvantage of this approach lies in the large overhead and the significant reduction in throughput [15]. For example, corresponding to the Partially Used Sub-Channelization (PUSC) method in the IEEE 802.16e (WiMAX) standard, each uplink PUSC tile consists of 4 adjacent subcarriers in frequency and 3 symbols in time, and 4 out of the 12 subcarrier-symbol combinations (i.e. approximately 33% of system resources) are for pilot [8].

The question that arises here is whether one can reduce the overhead inflicted by long CP. One possible answer to this question is to resort to dynamic resource allocation for mitigating MAI and improving user throughput. In this approach, the system parameters can be dynamically adjusted to adapt better to the wireless channel condition and the performance goals. This approach stems from the fact that MAI strongly depends on the assignment of frequency sub-bands to UTs, and is signif-

icantly reduced by suitable insertions of guards in the time and the frequency domains [9]. While guard in time or Guard Interval (GI) actually means the Cyclic Prefix (CP), a guard in frequency or Guard Band (GB) is an unmodulated frequency sub-band and assigned to no UTs. Therefore, exploiting the flexibility of resource allocation offered by OFDMA technique as well as the frequency and multi-user diversities is deemed to be promising to mitigate MAI and improve the UT performance.

In this report, we investigate the possibility of mitigating MAI and improvement of user throughputs in the OFDMA uplink through guard bands. We address the question whether using GB together with short CP can be advantageous at all compared with using long CP only. For this step we consider a scenario where a fixed width CP is used with fixed-width GB to mitigate MAI and improve UT throughput. Hence CP and GB is used in a static way. The performance of several combinations of CP and GB lengths are evaluated under different time and frequency offset. It is shown that short CP together with GBs can bring significant improvement in throughput compared to the mere usage of long CP.

The rest of this report is structured as follows. In Section 2, we describe the system model and main assumptions. Section 3 is dedicated to the study of the MAI mitigation with static assignment of GIs and GBs. The appendices present the proofs of main equations, and discuss the various numerical evaluation of preceding scenarios.

## Chapter 2.

# System Model

We consider one single cellular urban micro-cell consisting of one BS and  $M$  UTs. In the cell, OFDMA is used as data transmission scheme for downlink and uplink transmissions; the multiplexing method in use is Time Division Duplexing (TDD). The total available bandwidth  $B[H_z]$  is divided into  $N_{sca}$  subcarriers. Consequently, the subcarrier spacing is  $f_0 = B/N_{sca}$ , the OFDM symbol duration time takes a value of  $T_{sym} = 1/f_0$ , and the sampling interval equals  $T_{sam} = T_{sym}/N_{sca}$ . For each UT  $u$ , its relative time and frequency offsets relatively to BS are denoted by  $\tau_u$  and  $\Delta f_u$ , respectively. Let us assume that UTs always have data to send. Due to the frequency division multiple access (FDMA) scheme, UTs can take different unique subsets of total available subcarriers and send their data in parallel to BS through a multi-path fading channel. Let  $P_{k,u}^{TX}$  be the transmission power generated on subcarrier  $k$  by UT  $u$ . We assume that an equal and static transmission power  $P^{TX}$  for all subcarriers, it means  $P_{k,u}^{TX} = P^{TX}, \forall k, u$ . The number of OFDMA symbols in one uplink frame is denoted by  $N_{sym}$ .

The slow-fading channel in this paper is modeled reflecting path-loss, shadowing loss and multipath-loss. The selected models are listed as follows:

- Path-loss model: COST 231 Walfish-Ikegami with Non Line Of Sight (NLOS) (as recommended in [14]).
- Log-normal shadowing (as recommended in [14]).
- Multipath-loss: Clarke's model [13].
- The noise power is not explicitly modeled; the thermal noise density as adopted in the IEEE 802.16e standard [8] is used instead.

It is important to mention that during the downlink, in order to receive correctly the signals from BS, UTs needs to estimate the channel quality and the offsets. We assume these estimations are sent to BS. Consequently, BS presumably has perfect knowledge of the channel quality and time and frequency offset.

The signal arriving at BS is the superposition of attenuated, distorted, shifted in time and frequency constituents. In general, the received power on subcarrier  $k$  sent by UT  $u$  is  $P_{k,u}^{RX} = P^{TX} \times H_{k,u}$ , where  $H_{k,u}$  represents the  $u^{th}$  UT's average channel gain on subcarrier  $k$ . Similarly, the instantaneous Signal to Interference plus Noise Ratio (SINR)  $\gamma_{k,u}$  on the  $k^{th}$  subcarrier, which is uniquely assigned to UT  $u$  is given by

$$\gamma_{k,u} = \frac{P_{k,u}^{RX}}{\sigma^2 + \sum_{u' \neq u} \sum_{r \neq k} MAI_{k,u}^{r,u'}} \quad (2.1)$$

where  $\sigma^2$  is the noise power.  $MAI_{k,u}^{r,u'}$  is the average power of the MAI caused by subcarriers  $r$  of UT  $u'$  on subcarriers  $k$  of UT  $u$ . According to [9],  $MAI_{k,u}^{r,u'}$  is computed as follows:

$$MAI_{k,u}^{r,u'} = \frac{P_{r,u'}^{RX}}{N^2} \times \frac{A(k-r, \Delta f_{u'} - \Delta f_u, \tau_{u'} - \tau_u)}{\sin^2[\frac{\pi}{N}(k-r + \Delta f_{u'} - \Delta f_u)]} \quad (2.2)$$

where  $A(\cdot)$  is a function of the relative offset in time and frequency between UT  $u'$  and UT  $u$  as well as the distance between subcarriers  $k$  and  $r$ . The detailed calculation of  $A(\cdot)$  is presented in Appendix A.1 and it can be found in [9].

In order to avoid Inter-Symbol Interference (ISI), Cyclic Prefix (CP) is added in the time domain. To have a better insight, we assume CP consists of 2 parts, whose the lengths are  $v_1$  and  $v_2$ , respectively. The total OFDMA symbol duration time with CP is  $T = T_{sym} + v_1 + v_2$ . While  $v_2$  protects signals from the delay spread, the  $v_1$  part is to cope with two-way time offset caused by propagation delay and clock errors. Then ISI can be totally eliminated as long as the maximal time offset is smaller than  $\frac{1}{2}v_1$  and  $v_2$  is longer than maximal delay spread. It is recommended in IEEE 802.16m [8] to choose CP relatively long (e.g. 1/8 OFDMA symbol duration time). The MAI caused by skipping  $v_1$  as well as frequency offset is mitigated through the dynamic resource allocation.

In the allocation process,  $G$  adjacent subcarriers are grouped into one subchannel in order to reduce the overhead for allocation addresses and the searching space of the optimization problem. The total subchannel number is denoted by  $Q = N_{sca}/G$ .  $K_u \subset Q$  is the unique subset of subchannel assigned to UT  $u$ . In this paper, the size of subchannel is selected equal to the coherent bandwidth of the wireless channel. Hence, we have  $H_{k,u} = H_{i,u}$  and, thus,  $P_{k,u}^{RX} = P_{i,u}^{RX} = P^{TX} \times H_{i,u}$  for all subcarrier  $k$  of subchannel  $i$ . Consequently, we define  $\overline{MAI}_{i,u}^{j,u'}$  the average MAI caused by subchannel  $j$  of UT  $u'$  on subchannel  $i$  of UT  $u$  as follows:

$$\overline{MAI}_{i,u}^{j,u'} = \frac{1}{G} \sum_{k \in i} \sum_{r \in j} MAI_{k,u}^{r,u'} \quad (2.3)$$

We derive the average SINR of subchannel  $i$  of UT  $u$ :

$$\tilde{\gamma}_{i,u} = \frac{P_{i,u}^{RX}}{\sigma^2 + \sum_{u' \neq u} \sum_{j \neq i} \overline{MAI}_{i,u}^{j,u'}} \quad (2.4)$$

We use the Shannon capacity. The Shannon equation has to be changed in order to reflect the impact of the length of GI (i.e.  $v_1$  and  $v_2$ ) as follows:

$$C_{i,u} = \left( \frac{T_{sym}}{T_{sym} + v_1 + v_2} \right) f_0 \log_2(1 + SINR_{i,u}) \quad , \quad (2.5)$$

where  $T_{sym}$  denotes the symbol duration without CP and  $T_{sym} + v_1 + v_2$  means the symbol duration with CP.

## Chapter 3.

# Static assignment of Guard Bands and Guard Intervals

Given the previous model, we consider various scenarios with different lengths of GIs and GBs. For GI, the length of the part  $\nu_2$  is selected to equal the maximal delay spread, however  $\nu_1$  is changed for different scenarios. To observe the effect of the width of GI and GB, we consider a scenario, in which there are only two UTs in cell denoted by  $u_1$  and  $u_2$ . The UT  $u_1$  takes a number of subcarriers located in the center. There are these interfering subcarriers which cause MAI on neighbors (i.e. subcarriers of UT  $u_2$ ), which in turn leads to losses in SINR and, consequently, in user throughput. A flat channel is adopted, and the modified Shannon capacity as shown in Section 2 is used for the performance evaluation. The main characteristics of the scenario are shown in the Table 3.1. The detail of simulation results can be found in the Appendix.

From the formulation of MAI in [9], it can be seen that Guard Intervals can only protect user signals against time offsets and does not have any effect on frequency offsets. Particularly, with the existence of only time offset, the negative impact of MAI is fully mitigated when GI's width equals 2 times of the maximal time offset. However, with the existence of frequency offset with/without time offset, portion of MAI caused by frequency offset is totally independent of the GI's width.

Due to the nature of OFDM technique, CP is static and its length is fixed in advance. Hence it cannot dynamically deal with changes in the synchronization error during runtime. Consequently, CP might be either too long, which leads to throughput losses, or not long enough, which gives rise to the insufficient protection. Thus it is unlikely that the CP's width exactly matches the least requirement of OFDM system.

Another problem for GI is that it can not be parameterized differently for different sub-bands or UTs. In other words, the width is the same for all UTs although UTs might be differently asynchronous in time, hence, GI cannot cope with UTs' offset in time individually.

On the other hand, to reduce MAI, a number of subcarriers assigned to the badly-synchronized UT  $u$  can be taken away, left unmodulated and set as guard band. Obviously, turning off some interfering subcarriers reduces the MAI and, thus, improves SINR and rate of subcarriers of other UTs, no matter if MAI is caused by time offset or frequency offset or both. Therefore, GBs can be very flexibly set to help each UT individually against both time and frequency offsets. However, definite usage of GBs causes throughput loss for the UT which gives away subcarriers.

From the analysis in Appendix A.2, it can be seen that when frequency offset is zero, the length of GI has strong impact on the SINR. Especially, when GI equals two times of time offset, the negative impact on SINR caused by time offset is fully mitigated. However, when frequency offset is not zero, the length of GI has trivial impact on the SINR. It can be seen that when frequency offset increases, then the choice of GIs as before changes and eventually one has to choose the minimum length GI.

Table 3.1.: System parameters.

Parameters	Values
Number of UTs	2
Number of subcarriers	128
Number of interfering subcarriers of UT $u_1$ in centre	32
Avg. power per subcarrier	0 dBm
RSSI value of the flat wireless channel	-90 [dB]
Noise power	-133 [dBm]

This means that there is no fixed length for GI which works for almost all the scenarios.

On the other hand, one can properly choose an appropriate number of Guard Bands such that the effect of time and frequency offset on SINR is totally mitigated. GBs can protect user signal from frequency offset and/or time offset. However the mitigation of MAI and SINR improvement is obtained at the cost of resource wastage. When no frequency offset is present, it is still the best choice to use instead long CP. However one can still choose short CP and long GB to approach the optimal choice. As soon as there is a frequency offset in the system, using GBs can also improve the cell rate. For instance, when the time offset is present then using GBs with short CP is the best choice. It is important to mention that using cell rate as the criteria for throughput performance neglects the fairness between users. For instance, it can be seen that the maximum cell rate is achieved when the throughput of another user is set to zero. This shows that the fairness between users should be considered in the formulation of the optimization problem. Detailed discussion about the effect of GBs and GIs on MAI and throughput can be found in Appendix A.2.

## Chapter 4.

### Conclusion

It was shown that instead of using long CP and employing estimation techniques, one can use time and frequency guards together to mitigate the effect of MAI and avoid large overhead of long CP. When no frequency offset is present, it is better to use long CP rather than GBs however one can instead choose short CP and long GB to achieve the same performance. In case of frequency offset in the system, using GBs can also improve the cell rate. In presence of both frequency and time offset, using GBs with short CP achieves the best performance. Using cell rate as the criteria for throughput performance neglects the fairness between users and another formulation is needed to deal with this case.

## Appendix A.

## Appendix

### A.1. Calculation of MAI

Multiple access interference can be calculated as follows:

$$MAI_{k,u'}^{r,u} = \frac{P_{r,u}^{RX}}{N^2} \frac{A(k-r, \Delta f_u, \tau_u)}{\sin^2(\frac{\pi}{N}(k + \Delta f_u - r))} \quad (\text{A.1})$$

where  $A(k-r, \Delta f_u, \tau_u)$  is computed as follows

- Case 1:  $(-v_1/2 - v_2) > \tau_u \geq (-N + v_1/2)$ :

$$A = \sum_{l=0}^{N_u^p-1} \Omega_l^u \left[ \sin^2 \frac{\pi}{N} \left( l - \tau_u - \frac{v_1}{2} - v_2 \right) (\Delta f_u + k - r) \right. \\ \left. + \sin^2 \frac{\pi}{N} \left( l - \tau_u - \frac{v_1}{2} - v_2 - N \right) (\Delta f_u + k - r) \right]$$

- Case 2:  $(N_u^p - 1 - v_1/2 - v_2) > \tau_u \geq (-v_1/2 - v_2)$ :

$$A = \sin^2(\pi \Delta f_u) \sum_{l=0}^{v-|\tau_u|} \Omega_l^u \quad (\text{A.2}) \\ + \sum_{l=v-|\tau_u|+1}^{N_u^p-1} \Omega_l^u \left[ \sin^2 \frac{\pi}{N} \left( l - \tau_u - \frac{v_1}{2} - v_2 \right) (\Delta f_u + k - r) \right. \\ \left. + \sin^2 \frac{\pi}{N} \left( l - \tau_u - \frac{v_1}{2} - v_2 - N \right) (\Delta f_u + k - r) \right]$$

- else if  $-v_1/2 \geq \tau_u \geq (N_u^p - 1 - v_1/2 - v_2)$  then:

$$A = \sin^2(\pi \Delta f_u) \sum_{l=0}^{N_u^p-1} \Omega_l^u$$

- else if  $N_u^p - 1 + v_1/2 \geq \tau_u > v_1/2$  then:

$$A = \sin^2(\pi \Delta f_u) \sum_{l=|\tau_u|+1}^{N_u^p-1} \Omega_l^u \quad (\text{A.3}) \\ + \sum_{l=0}^{|\tau_u|} \Omega_l^u \left[ \sin^2 \frac{\pi}{N} \left( l - \tau_u + \frac{v_1}{2} \right) (\Delta f_u + k - r) \right. \\ \left. + \sin^2 \frac{\pi}{N} \left( l - \tau_u + \frac{v_1}{2} + N \right) (\Delta f_u + k - r) \right]$$

- else if  $(N + v_1/2) \geq \tau_u > (-N_u^p - 1 + v_1/2)$  then:

$$A = \sum_{l=0}^{N_u^p-1} \Omega_l^u \left[ \sin^2 \frac{\pi}{N} \left( l - \tau_u + \frac{v_1}{2} \right) (\Delta f_u + k - r) \right. \\ \left. + \sin^2 \frac{\pi}{N} \left( l - \tau_u + \frac{v_1}{2} + N \right) (\Delta f_u + k - r) \right]$$

where the wireless channel between the  $u^{th}$  UT and BS assumably consists of  $N_u^p$  resolvable paths .

## A.2. Numerical Performance Evaluation for Static GBs and GIs

We simulate a simple scenario on the OMNeT++ simulation platform (omnetpp.org). We consider only 2 UTs in cell denoted by  $u_1$  and  $u_2$ . The UT  $u_1$  takes a number of subcarriers locating at the center, then these interfering subcarriers cause MAI on the neighbors (i.e. subcarriers of UT  $u_2$ ), which in turn leads to losses in SINR and, consequently, in user rate. Moreover, a flat channel is adopted, and the Shannon capacity as shown in Section 2 is used for the performance evaluation. The important values are shown in the Table 3.1.

The impact of using only CP on SINR are then shown in Figures B.2, B.1, and B.3. When frequency offset is zero, i.e. Figure B.1, the length of GI has strong impact on the SINR. Especially, when GI equals two times of time offset, the negative impact on SINR caused by time offset is fully mitigated. This choice leads to the best cell rate as it can be seen in Figures B.8 and B.7. Therefore choosing GI equal to two times of time offset would be the best choice when no frequency offset is present.

However, when frequency offset is not zero, the length of GI has no impact on the SINR as it can be seen in the Figure B.2. Moreover it is not always good to choose GI equal to two times of time offset. In Figure B.7, it can be seen that when frequency offset increases, then the choice of best GIs changes and eventually one has to choose the minimum length GI.

To deal with the frequency offset, one can use GBs. The impact of using only GBs is presented in Figures B.5, B.4 and B.6, respectively. It can be seen that one can properly choose number Guard Bands such that the effect of time and frequency offset on SINR is totally mitigated. GBs can protect user signal from frequency offset and/or time offset. However the mitigation of MAI and SINR improvement is obtained at the cost of resource wastage. It can be seen in B.8 that when no frequency offset is present, it is still better to use long CP rather than GBs however one can still choose short CP and long GB to approach the optimal choice. But as soon as there is a frequency offset in the system, using GBs can also improve the cell rate. For instance, when the time offset is present then using GBs with short CP is the best choice as it can be seen in B.9. In presence of both frequency and time offset, one can see in Figure B.10 that using GBs with short CP is still the best choice.

It is important to mention that using cell rate as the criteria for throughput performance neglects the fairness between users. For instance, in Figure B.11, it can be seen that the maximum cell rate is achieved when the throughput of another user is set to zero. This shows that the fairness between users should be considered in the formulation of the optimization problem.

# **Appendix B.**

## **List of Figures**

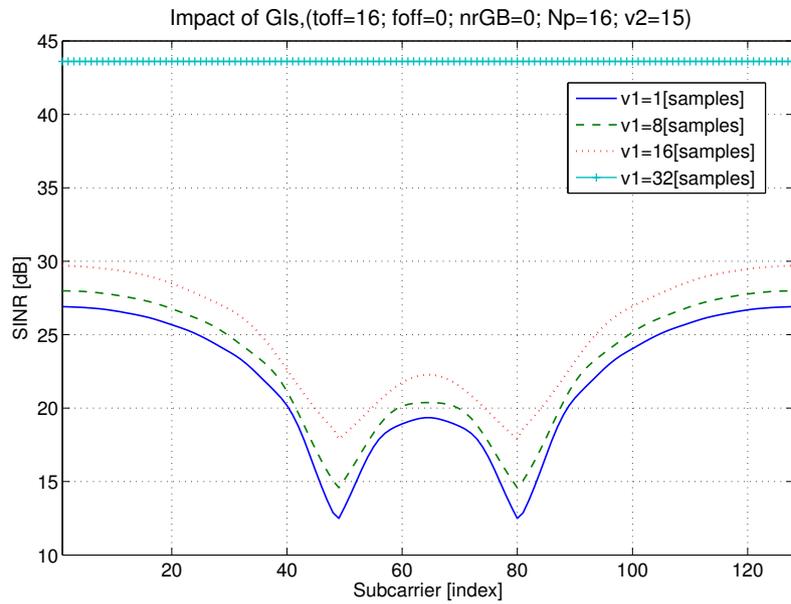


Figure B.1.: GI with Time offset only

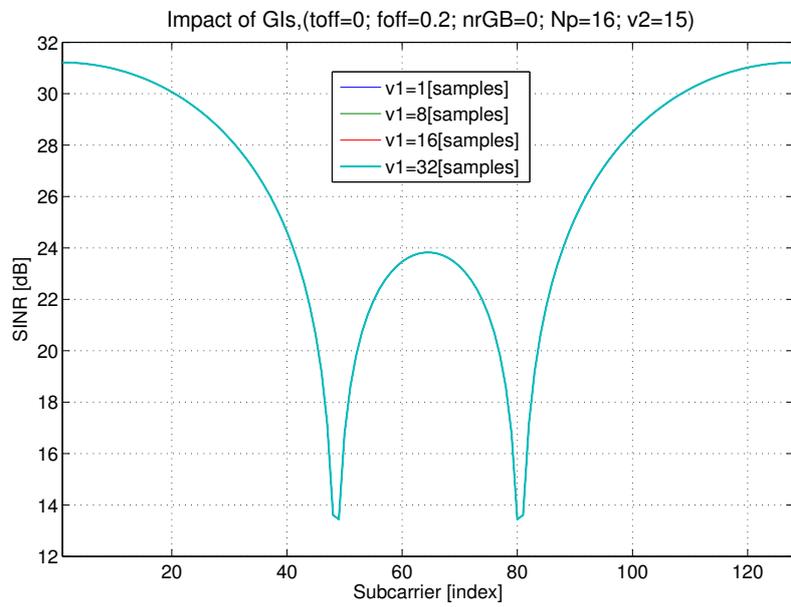


Figure B.2.: GI with Frequency offset only

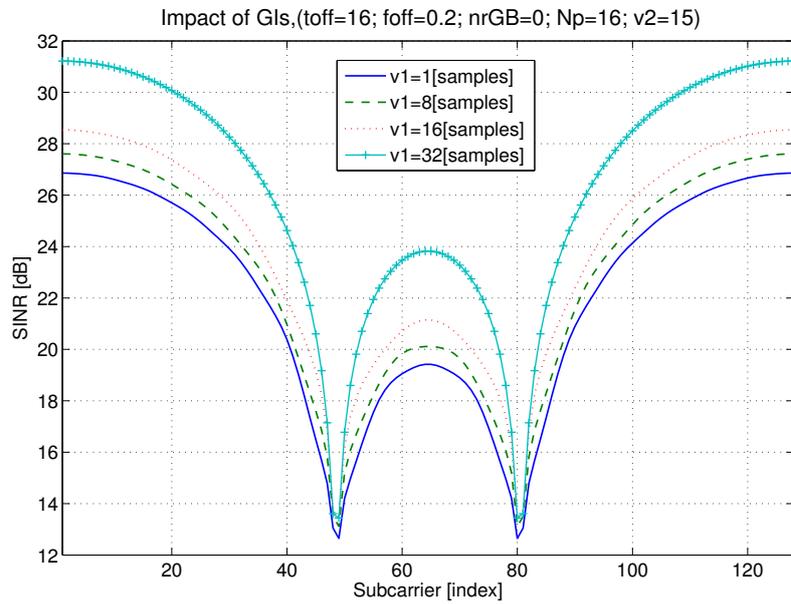


Figure B.3.: GI with Time, frequency offset

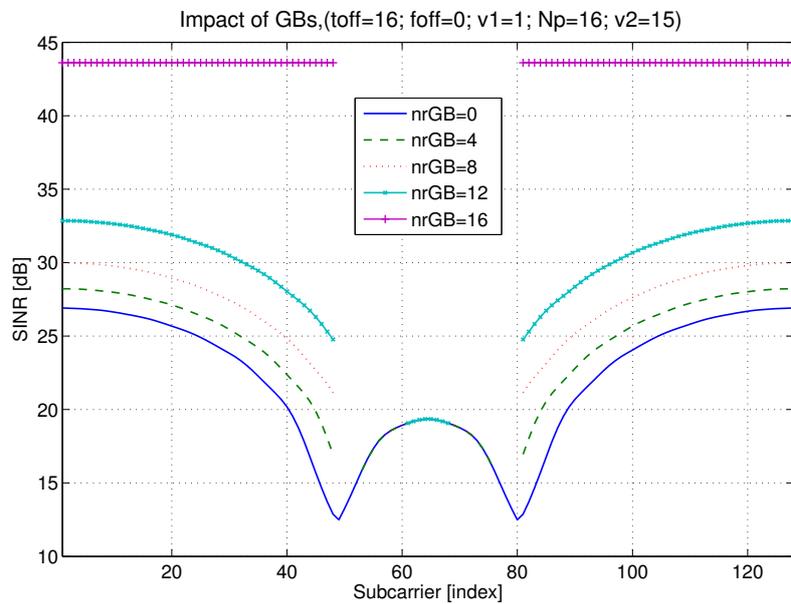


Figure B.4.: GB with Time offset only

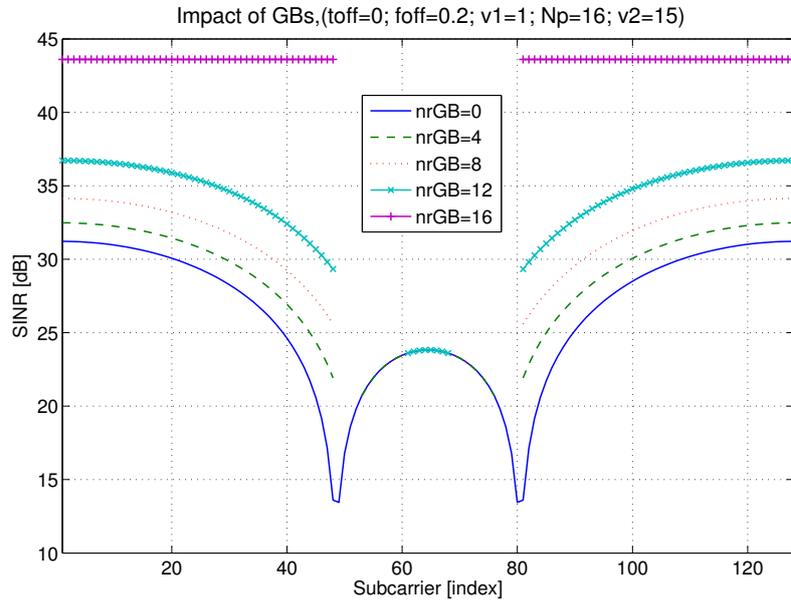


Figure B.5.: GB with Frequency offset only

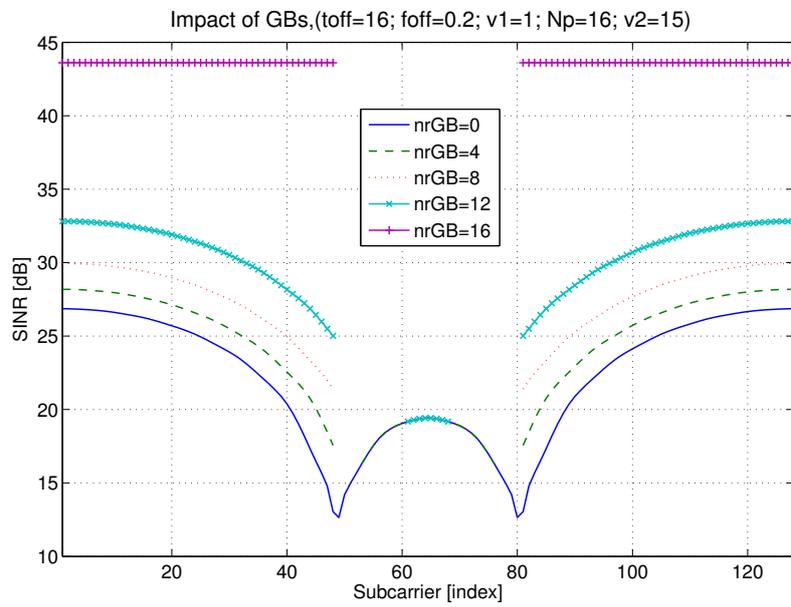


Figure B.6.: GB with Time, frequency offset

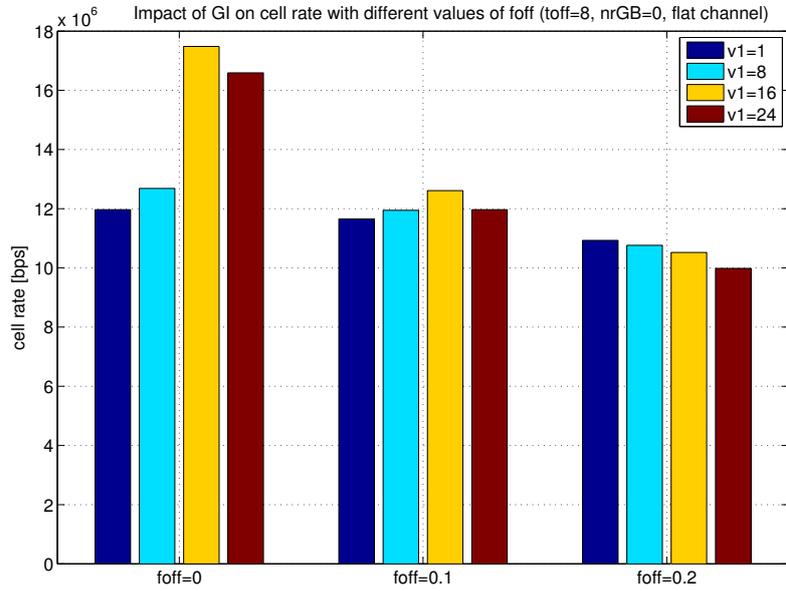


Figure B.7.: Impact of GI on cell capacity with different values of frequency offset (nrGB=0, flat channel).

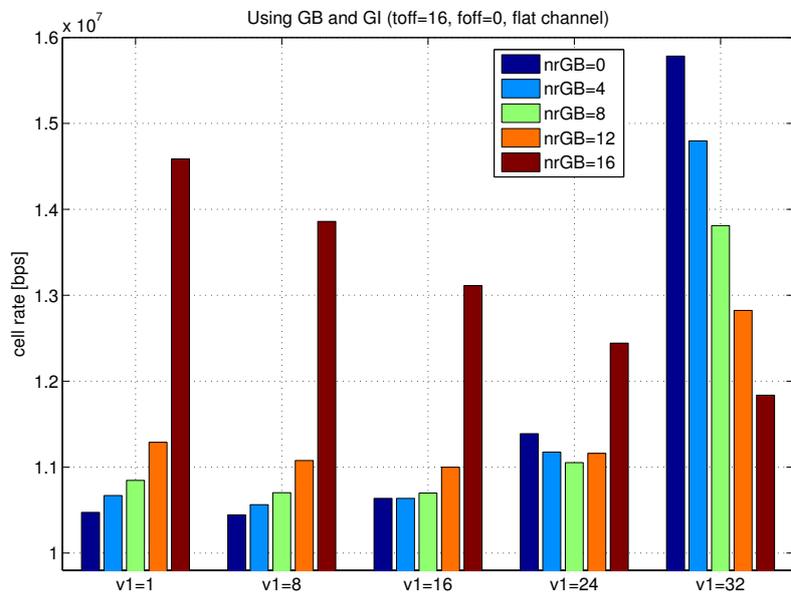


Figure B.8.: Cell capacity with different GI and GB values (toff=16, fo=0, flat channel).

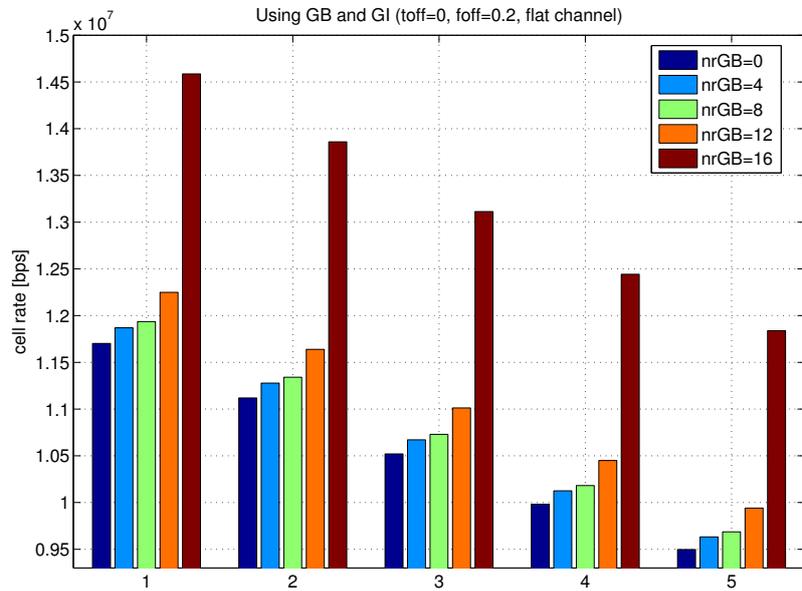


Figure B.9.: Cell capacity with different GI and GB values (toff=0, foff=0.2, flat channel).

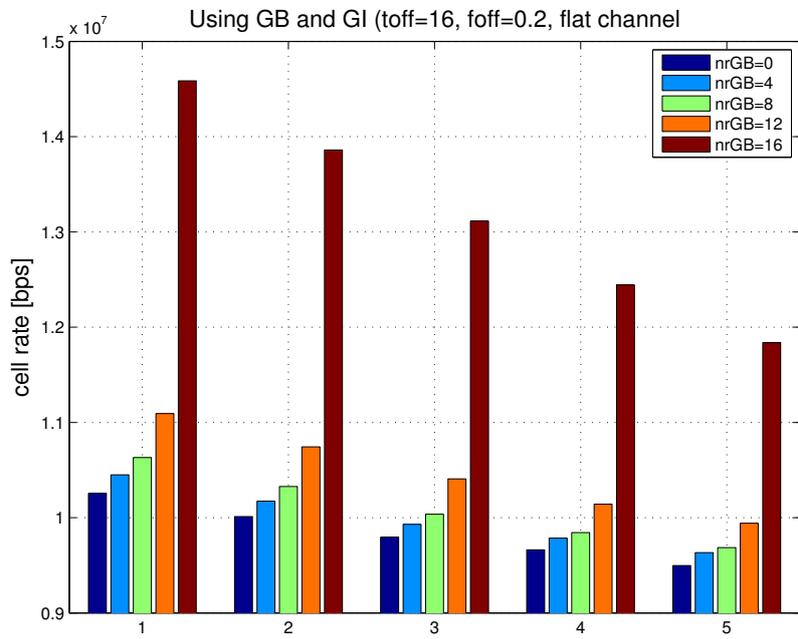


Figure B.10.: Cell capacity with different GI and GB values (toff=16, foff=0.2, flat channel).

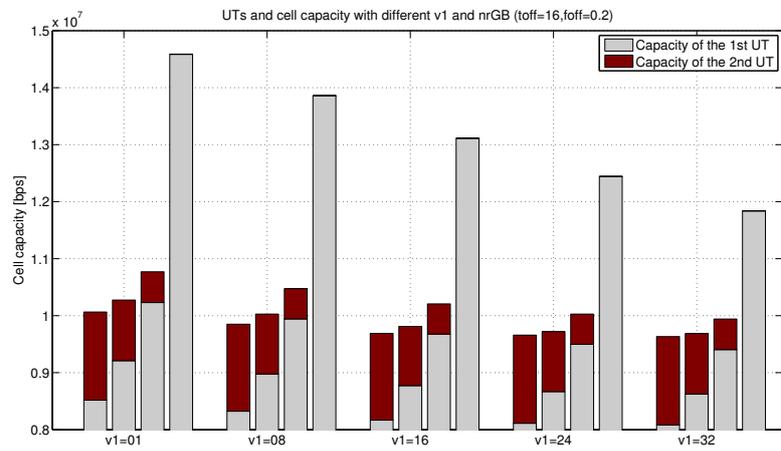


Figure B.11.: Cell and UTs' capacity (toff=16, foff=0.2, flat channel).

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