Selective Multi-User Cooperative Rate Splitting Assisted by Reconfigurable Intelligent Surface

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***Abstract*—Rate splitting multiple access (RSMA) has emerged as a powerful multiple access technique for wireless communications. In this paper, RSMA is integrated with cooperative relaying and reconfigurable intelligent surfaces (RIS) to enhance the spectral efficiency of multi-user communication. We propose a selective approach that chose some users to decode the common stream using successive interference cancellation (SIC) and treat the private streams as noise. While the other users do not decode the common stream, they treat it as noise. The selection is based on the condition of the users' channel. The user with the worst channel condition is selected to treat the common stream as noise. The performance of the proposed system is measured in terms of Max-Min rate of the users. Alternative optimization (AO) is used to jointly optimize the time slot allocation, precoding matrix, common rate allocation, and phase shifts of RIS in the direct and cooperative phase iteratively. Numerical results show that the proposed selective cooperative rate splitting assisted by RIS (CRS-RIS) improves significantly the Max-Min rate when compared with other existing systems.**

***Index Terms*—Sixth generation, mobile communications, rate splitting multiple access, reconfigurable intelligent surface, cooperative communication**

I. Introduction

Nowadays, the sixth-generation mobile communications (6G) and beyond are attracting momentous attention from academia and industry due to their ability in enabling the internet of everything, providing services with higher throughput, ultra-reliability, heterogeneous quality of service, massive connectivity and ultra-reliable low latency [1]. 6G should support multi-user communication, which require an efficient techniques to manage the wireless resources and the interference. One of these techniques is rate splitting multiple access (RSMA) [2].

RSMA is a promising multiple access technique for non-orthogonal transmission and interference management. The concept of RSMA is that the message of each user is split into two parts: common part and private one, then the common part are encoded into common streams, while the private parts are encoded into private streams. Initially, the common streams should be decoded by all users, and then each user starts to decode its own private stream. This enables the capability of partially decoding the interference and partially treating the interference as noise, also RSMA can unifies the existing multiple access techniques [2]. It should be considered that the achievable rate of common stream is limited by the worst user's rate, which may affect the system performance. In order to enhance the rate of the common streams, cooperative communication is integrated with RSMA, which is known as cooperative rate splitting (CRS). In CRS the user with strong channel can act as a relay to assist the base station (BS) in transmitting the common stream to the users with weak channel conditions [3].

Furthermore, one of the most important technologies in 6G that can improve system performance significantly are reconfigurable intelligent surfaces (RIS). Conceptually, an RIS is an easily deployed-planar surface that consists of large number of reflecting elements, these reflecting elements have the ability to adjust the amplitude and the phase shift of the incident signal, hence they can reconfigure the wireless propagation environment. RIS can improve the spectral efficiency as they can create a virtual line-of-sight between transmitter and receiver, also they can null the interference by adjusting the amplitude and phase shifts of reflecting elements. Due to the mentioned benefits of both RSMA and RIS, the integration of both technologies is expected to improve the system performance even more.

For the best of our knowledge, investigating the performance of selective multi-user cooperative rate splitting multiple access assisted by RIS is not deeply analyzed before. In this paper, we integrate RSMA with RIS and cooperative relaying to earn the benefits of these technologies. Instead of using RSMA for all users, we propose a downlink selective CRS scheme, where some users are selected to decode the common stream using SIC while the other users decode only their private stream by treating the common stream and private streams of other users as noise.

Our main contributions can be summarized as follows:

- We integrate the selective RSMA with user relaying and RIS to improve the maximum minimum rate (Max-Min rate) of multi-user mobile terminals.
- We formulate an optimization problem that maximizes the minimum rate of the users, by jointly optimizing the time slot allocation, transmit percoding matrix, the common rate allocation, and phase shifts of RIS in the direct transmission phase and in the cooperative one.
- The optimization problem is divided into sub-problems to itertively optimize the precoding matrix and common rate allocation, the phase shifts in the direct transmission phase, and the phase shifts in the cooperative transmission phase, receptively.

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II. Related Work

There is a rich body of literature that showed that RSMA outperforms the other multiple access techniques (MA) and improves the system performance significantly. In [4], it shows that RSMA outperforms non-orthogonal multiple access (NOMA), frequency division multiple access (FDMA), and time division multiple access (TDMA) in terms of sum-rate. The authors in [5] used successive convex approximation (SCA) to maximize the energy efficiency, and make a comparison between energy efficiency of RSMA and that of NOMA and SDMA. Numerical results ensures that RSMA has better performance than the the other MA techniques. Furthermore, Latency can be enhanced by using RSMA when compared to the other MA, and this is shown in [6]. In [4], [5], and [6], they only consider RSMA systems without integrating them with any other technologies, which does not earn the full benefit of the integrated system.

Some literature integrate RSMA with RIS to improve the system performance [7]-[13]. In [7], the sum rate is maximized by selecting the reflecting coefficients at the RIS and designing beamformers at the BS under the constraints of power at the base station (BS), quality of service (QoS) at each user and finite resolution at the RIS, while in [8], the sum rate is maximized by optimizing power allocation and beamforming design using successive convex approximation, Riemannian manifold and fractional programming techniques. The spectral efficiency was also improved by optimizing the transmit beamforming and the phase matrix of passive and active RIS respectively [9], [10]. Also, the system performance is enhanced in term of outage probability [11], energy efficiency [12] and the secure transmission [13].

Cooperative rate splitting (CRS) is introduced [14]-[16] to improve the performance of the system. In [14], it was shown that CRS outperforms the non-cooperative RSMA (non-CRS) and the other existing MA techniques. In [15], the authors used two power allocation algorithms based on successive-convex-approximation (SCA) and geometric-programming (GP) in order to maximize the minimum rate of two users. Numerical results showed that CRS has better performance when compared with non-CRS and cooperative NOMA. While in [16], the authors used the full-duplex (FD) CRS in order to enhance minimum achievable rate.

Furthermore, the attention is directed towards CRS assisted by RIS (CRS-RIS) to show the high capability of RSMA-RIS. In [3], the energy consumption is enhanced even more when compared CRS-RIS with RSMA-RIS and the other MA techniques. In [17], the minimum rate is maximized by optimizing the transmit beamforming, common rate allocation and RIS phases using alternative optimization algorithm. From the above survey, it is noted that selective CRS-RIS in case of multi-user is not investigated deeply. In this paper, we consider such system where some users are selected to use RSMA, while the others not apply it. The motivation behind this is to overcome the limitation of the system performance due to the low rate of the users with weak channel conditions.

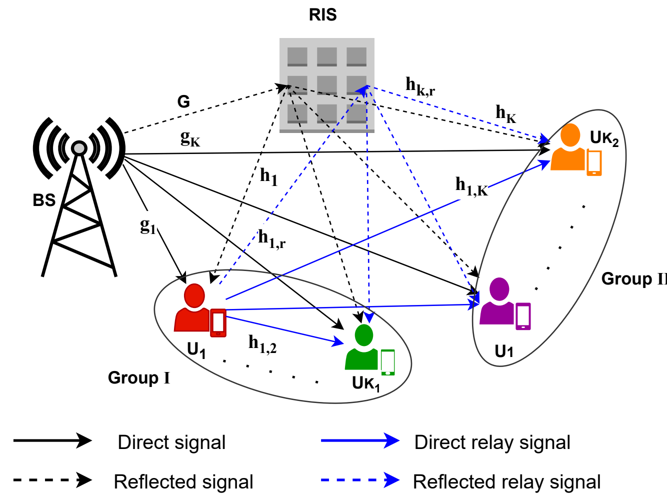

Fig. 1: Selective multi-user CRS-RIS

III. System Model and User Grouping

A. System Model

We consider a downlink RIS-assisted CRS wireless multi-user communication system. As shown in Fig. 1, the system consists of one base station (BS) equipped with N_t transmit antennas that serves K users with single antenna, and one RIS with N passive reflecting elements. The system follows 1-layer RSMA principle. The proposed CRS-RIS adapts the selection strategy, where the K users are divided into two groups denoted by group I and group II. Group I contains K_1 users, while group II contains K_2 users. The users in group I use the principle of RSMA and called RS-users. While the users in group II do not use RSMA principle. Thus, $K_1 \cup K_2$ = K and $K_1 \cap K_2$ = $\emptyset$. One of the users acts as a half-duplex (HD) relay to forward the common stream to rest of the users. Selection of relay and the users in group I and in group II will be explained in the next section. The channel between the BS and the RIS is denoted by $\mathbf{G} \in \mathbb{C}^{N \times N_t}$, the channel between the BS and user-k is denoted by $\mathbf{g}_k \in \mathbb{C}^{N_t \times 1}$, the channel between RIS and user-k in first time slot is denoted by $\mathbf{h}_k \in \mathbb{C}^{N \times 1}$, the channel between RIS and user-k in second time slot is denoted by $\mathbf{h}_{k,r} \in \mathbb{C}^{N \times 1}$, and the channel between two different users is denoted by h_{ij} where $i \neq j$, and $i, j \in K$. The channels are modeled with Rayleigh fading model.

The transmission process is divided into two time slots. The first time slot is the direct transmission phase and the second one is the cooperative transmission phase. During the first time slot, the BS transmits signals to all users based on 1-layer RSMA principle, and the transmitted signal is reflected by RIS to the users, while in the second time slot the chosen user relay (U_A) re-transmits the common stream to the rest of users, meanwhile the BS remains silent. The fraction of time allocated to the first slot is α $(0 < \alpha < 1)$, while the rest $(1 - \alpha)$ is allocated to the second slot.

B. Selection of RS-Users and the Relay

Selection of the users perform rate splitting (RS) is based on the users channel condition. The users with the best channel condition use the RSMA principle. That is, these users decode the common stream using SIC and treat the private stream as noise. The users of the worst channel conditions treat the common stream as noise. They do not decode it. The idea behind this is that in order to achieve high rate of the RSMA, the common stream should be decoded correctly. This is achieved by assigning the users with good channel quality to group I, to use RSMA. The other users are assigned to group II.

One of the users in group I, is chosen to act as a relay to decode the common stream and forward it to the rest of the users. The user with the best channel condition among the users of group I is chosen to act as a relay. We assume that all channels are perfectly known to receivers.

IV. Problem Formulation

A. Direct Transmission Phase

The message of user-k, W_k, in group I is split into two parts: common part $W_{c,k}$ and private part $W_{p,k}$. All the common parts of the users in group I are combined together and encoded into common stream s_0 using a common codebook, while the private parts are encoded independently and transformed to private stream s_k. The message of the users in group II do not have the common part, only the private one $W_{p,k}$, so they are directly encoded and transformed to private stream s_k. The common stream is intended for all users in group I but the private streams are intended for their corresponding users. Then, the signal transmitted by BS can be written as:

$$\mathbf{x} = \sum_{k=0}^{K} \mathbf{p}_k \mathbf{s}_k \tag{1}$$

where $\mathbf{P} \in \mathbb{C}^{N_t \times (K+1)}$ is the precoding matrix and $\mathbf{p}_k \in \mathbb{C}^{N_t \times 1}$.

Assuming that $\mathbb{E}[\mathbf{s}\mathbf{s}^H] = \mathbf{I}$ and the transmit power constraint is $tr(\mathbf{P}\mathbf{P}^H) \leq P_t$, P_t is the maximum transmit power of the BS. Then, the received signal at user-k in first time slot is given by:

$$y_k^{(1)} = (\mathbf{g}_k^H + \mathbf{h}_k^H \mathbf{\Phi}_{(\mathbf{1})} \mathbf{G})\mathbf{x} + n, \forall k \in K \tag{2}$$

where $\mathbf{\Phi}_{(\mathbf{1})}$ = diag $(e^{j\phi_1 (1)}, e^{j\phi_2 (1)}, ..., e^{j\phi_N (1)})$ is the reflection matrix of RIS in the first time slot, and n is complex additive white Gaussian noise (AWGN) at the users with power spectral density donated by N_0. When user-k in group I receives the transmitted signal, it first decodes the common stream s_0 by treating the private streams as interference. Then, the achievable rate of decoding the common stream at user-k is

$$c_k^{(1)} = \alpha \log_2(1+\frac{\| (\mathrm{g}_k^H + \mathbf{h}_k^H \mathbf{\Phi}_{(\mathbf{1})} \mathbf{G})\mathbf{p}_0 \|^2}{\sum_{i=1}^{K} \| (\mathrm{g}_k^H + \mathbf{h}_k^H \mathbf{\Phi}_{(\mathbf{1})} \mathbf{G})\mathbf{p}_i \|^2 + N_0}), \tag{3}$$

for $\forall k \in K_1$, after removing the decoded common stream, the rate of decoding the private stream at user-k in group I is

$$r_{k,1}^{(1)} = \alpha \log_2(1+\frac{\| (\mathrm{g}_k^H + \mathbf{h}_k^H \mathbf{\Phi}_{(\mathbf{1})} \mathbf{G})\mathbf{p}_k \|^2}{\sum_{\substack{i=1 \\ i\neq k}}^{K} \| (\mathrm{g}_k^H + \mathbf{h}_k^H \mathbf{\Phi}_{(\mathbf{1})} \mathbf{G})\mathbf{p}_i \|^2 + N_0}), \tag{4}$$

for $\forall k \in K_1$, while user-k in group II receives the transmitted signal, and directly decodes the private stream by treating the common stream and the other private streams as noise, thus, the rate of decoding the private stream at user-k in group II is

$$r_{k,2}^{(1)} = \alpha \log_2(1+\frac{\| (\mathrm{g}_k^H + \mathbf{h}_k^H \mathbf{\Phi}_{(\mathbf{1})} \mathbf{G})\mathbf{p}_k \|^2}{\sum_{\substack{i=0 \\ i\neq k}}^{K} \| (\mathrm{g}_k^H + \mathbf{h}_k^H \mathbf{\Phi}_{(\mathbf{1})} \mathbf{G})\mathbf{p}_i \|^2 + N_0}) \tag{5}$$

for $\forall k \in K_2$.

B. Cooperative Transmission Phase

In this phase, the BS is silent and the relaying user forwards the common stream s_0 to the other users in group I using decode-and-forward (DF) approach. The users in group II receives nothing in this slot, since they not apply the RSMA principle. Thus, this phase is dedicated only to RS-users in group I. The relaying user forwards the stream with transmit power P_r and a phase matrix for the second time slot $\mathbf{\Phi}_{(\mathbf{2})}$. Then, the received signal at the non-relaying users is

$$y_k^{(2)} = (\mathrm{h}_{A,k} + \mathbf{h}_{k,r}^H \mathbf{\Phi}_{(\mathbf{2})} \mathbf{h}_{A,r})\sqrt{P_r} s_0 + n, \tag{6}$$

where $k \in K_1$, and $k \neq A$. $\mathrm{h}_{A,k}$ is the channel between user-k and the relay user (U_A), and $\mathbf{h}_{A,r}$ is the channel between RIS and the relay user (U_A).The rate of decoding the common stream for non-relaying users in the second time slot is given by

$$c_k^{(2)} = (1-\alpha) \log_2(1+\frac{P_r \| (\mathrm{h}_{A,k} + \mathbf{h}_{k,r}^H \mathbf{\Phi}_{(\mathbf{2})} \mathbf{h}_{A,r}) \|^2}{N_0}), \tag{7}$$

for $k \in K_1$, and $k \neq A$, then the users combine the decoded common stream from the two time slots. In order to guarantee that all users can decode the common stream s_0 successfully, the achievable rate to decode the common stream should be

$$R_c = \min(c_1^{(1)}, c_2^{(1)} + c_2^{(2)}, ..., c_{K_1}^{(1)} + c_{K_1}^{(2)}), \tag{8}$$

where R_c is shared by all users, as the common stream contains information from each user, thus it should satisfy

$$\sum_{i=1}^{K_1} a_i \leq R_c \tag{9}$$

where a_i is the portion of the common stream allocated to each user. Hence, the total achievable rate of user-k in group K_1 is

$$R_{k,tot} = r_{k,1}^{(1)} + a_k, \forall k \in K_1, \tag{10}$$

while the total achievable rate of user-k in group II is

$$R_{k,tot} = r_{k,2}^{(1)}, \forall k \in K_2. \tag{11}$$

The objective of this paper is to maximize the minimum rate (Max-Min rate) of all users, this can be done by jointly optimizing the time slot allocated to each slot α, the precoding matrix $\mathbf{P}$, the rate portion of common stream for each user $\mathbf{a}$, and the phase shifts of reflection matrix at the first $\mathbf{\Phi_{(1)}}$ and the second time slot $\mathbf{\Phi_{(2)}}$. The optimization problem can be formulated as the following:

$$\max_{\alpha,\mathbf{P},\mathbf{a},\mathbf{\Phi_{(1)}},\mathbf{\Phi_{(2)}}} \min\ R_{k,tot} \tag{12}$$

$$s.t.: \sum_{i=1}^{K_1} a_i \leq R_c, \tag{12a}$$

$$a_k \geq 0, \forall k \in K_1, \tag{12b}$$

$$\mathbf{\Phi}_{(j)} = \text{diag}(e^{j\phi_1{}^{(j)}}, e^{j\phi_2{}^{(j)}}, ..., e^{j\phi_N{}^{(j)}}), j \in \{1,2\}, \tag{12c}$$

$$\phi_n^{(j)} \in [0, 2\pi), \forall n \in N, j \in \{1,2\}, \tag{12d}$$

$$tr(\mathbf{P}\mathbf{P}^H) \leq P_t, \tag{12e}$$

where constraint (12a) the range of the possible values that the phase shifts can take can be shown in constraint (12d), while (12e) refers to the power budget at the BS. This optimization problem is non convex problem. Thus, this problem is divided into sub-problems which will be solved separately and iteratively to optimize the optimization variables till we reach the optimal value. This method is called alternative optimization algorithm.

V. Optimization Technique

Due to the non-convexity of (12), we divide the optimization problem into four parts. The first part is to optimize the time slot allocation α, the second part is to optimize the precoding matrix and the common rate allocation $\mathbf{a}$, the third part is to optimize the phase shifts of RIS in first time slot $\mathbf{\Phi_{(1)}}$, and finally the last sub-problem is to optimize the phase shifts of RIS in second time slot $\mathbf{\Phi_{(2)}}$. So first, optimizing the time slot allocation α is done using exhaustive research [3]. While the other optimization sub-problems can be described in the following subsections.

A. Joint Precoding Matrix and Common Rate Allocation Optimization

After optimizing the time slot allocation and for given value of phase shifts of RIS in both time slots, we focus on optimizing both the precoding matrix and the common rate. The channel expression in the first time slot, can be written as $\hat{\mathbf{g}}_k = \mathbf{g}_k + \mathbf{G}^H \mathbf{\Phi}_{(1)}^H \mathbf{h}_k$. Then (12), can be reformulated as the following

$$\max_{\mathbf{P},\mathbf{a},v} \quad v \tag{13}$$

$$s.t.: r_{k,1}^{(1)} + a_k \geq v, \forall k \in K_1, \tag{13a}$$

$$r_{k,2}^{(1)} \geq v, \forall k \in K_2, \tag{13b}$$

$$(12a), (12b), (12e),$$

where v donates to the minimum rate of all users. We can notice that (13) is still non convex due to constraints (12a), (13a) and (13b), so we introduce slack variables to solve this non-convexity. Let $\boldsymbol{\tau} = [\tau_1, ..., \tau_{K_1}]$, and $\boldsymbol{\tau_c} = [\tau_{c,1}, ..., \tau_{c,K_1}]$, denotes the signal-to-interference-plus-noise ratio (SINR) vector for the private and common streams of users in group I respectively. Let $\boldsymbol{\tau_s} = [\tau_{s,1}, ..., \tau_{s,K_2}]$, denotes the signal-to-interference-plus-noise ratio (SINR) vector for the private streams of users in group II. Thus, (12a) can be reformulated as

$$\sum_{i=1}^{K_1} a_i \leq \alpha \log_2(1 + \tau_{c,A}), \tag{14}$$

$$\sum_{i=1}^{K_1} a_i \leq \alpha \log_2(1+\tau_{c,k}) + (1-\alpha)\log_2(1+c_k^{(2)}), k \in K_1, k \neq A, \tag{15}$$

The relation between the introduced slack variable $\tau_{c,k}$ and the SINR is written as

$$\frac{\| \hat{\mathbf{g}}_k^H \mathbf{p}_0 \|^2}{\sum_{i=1}^{K} \| \hat{\mathbf{g}}_k^H \mathbf{p}_i \|^2 + N_0} \geq \tau_{c,k}, \forall k \in K_1, \tag{16}$$

Now, (16) is still non-convex, hence a new slack variable $\boldsymbol{\mu_c} = [\mu_{c,1}, ..., \mu_{c,K_1}]$, is introduced which denotes the interference-plus-noise term for the common stream of users in group I. Therefore, (16) can be written as

$$\frac{\| \hat{\mathbf{g}}_k^H \mathbf{p}_0 \|^2}{\mu_{c,k}} \geq \tau_{c,k}, \forall k \in K_1, \tag{17}$$

where $\mu_{c,k}$ is the denominator of SINR equation in (16), thus its expression is given by

$$\mu_{c,k} \geq \sum_{i=1}^{K} \| \hat{\mathbf{g}}_k^H \mathbf{p}_i \|^2 + N_0, \forall k \in K_1, \tag{18}$$

While for constraint (13a), we introduce slack variable $\boldsymbol{\mu} = [\mu_1, ..., \mu_{K_1}]$, which denotes the interference-plus noise term for the private stream of users in group I. Then, (13a) can be represented as follows

$$\alpha \log_2(1 + \tau_k) + a_k \geq v, \forall k \in K_1, \tag{19}$$

$$\frac{\| \hat{\mathbf{g}}_k^H \mathbf{p}_k \|^2}{\mu_k} \geq \tau_k, \forall k \in K_1, \tag{20}$$

$$\mu_k \geq \sum_{\substack{i=1 \\ i \neq k}}^{K} \| \hat{\mathbf{g}}_k^H \mathbf{p}_i \|^2 + N_0, \forall k \in K_1, \tag{21}$$

Similarly, for constraint (13b), we introduce slack variable $\boldsymbol{\mu_s} = [\mu_{s,1}, ..., \mu_{s,K_2}]$, which denotes the interference-plus noise term for the private stream of users in group II. Then, (13b) can be represented as follows

$$\alpha \log_2(1 + \tau_{s,k}) \geq v, \forall k \in K_2, \tag{22}$$

$$\frac{\| \hat{\mathbf{g}}_k^H \mathbf{p}_k \|^2}{\mu_{s,k}} \geq \tau_{s,k}, \forall k \in K_2, \tag{23}$$

$$\mu_{s,k} \geq \sum_{\substack{i=0 \\ i \neq k}}^{K} \parallel \hat{\mathbf{g}}_k^H \mathbf{p}_i \parallel^2 + N_0, \forall k \in K_2, \tag{24}$$

However, (17), (20) and (23) are still non-convex; but they follow a generic form as $f(x,y) = \frac{\|y\|^2}{x}$, $\forall y \in \mathbb{C}, \forall x \in \mathbb{R}^+$, that can be approximated using a lower bounded concave approximation as mentioned in [3]. The function $f(x,y)$ can be approximated on point x^m,y^m in order to solve non-convexity in iterative manner. Then, the approximated generic equation is given by

$$f(x,y) \geq F(x,y;x^m,y^m) = \frac{2\mathcal{R}\{y^{(m)*}y\}}{x^m} - \frac{\parallel y^m \parallel^2}{(x^m)^2}x, \tag{25}$$

Using (25), equations (17), (20) and (23) can be approximated by (26), (27) and (28), respectively:

$$\frac{2\mathcal{R}\{(\mathbf{p}_0^m)^H \hat{\mathbf{g}}_k \hat{\mathbf{g}}_k^H \mathbf{p}_0\}}{\mu_{c,k}^m} - \frac{\parallel \hat{\mathbf{g}}_k^H \mathbf{p}_0^m \parallel^2 \mu_{c,k}}{(\mu_{c,k}^m)^2} - \geq \tau_{c,k}, \forall k \in K_1, \tag{26}$$

$$\frac{2\mathcal{R}\{(\mathbf{p}_k^m)^H \hat{\mathbf{g}}_k \hat{\mathbf{g}}_k^H \mathbf{p}_k\}}{\mu_k^m} - \frac{\parallel \hat{\mathbf{g}}_k^H \mathbf{p}_k^m \parallel^2 \mu_k}{(\mu_k^m)^2} - \geq \tau_k, \forall k \in K_1, \tag{27}$$

$$\frac{2\mathcal{R}\{(\mathbf{p}_k^m)^H \hat{\mathbf{g}}_k \hat{\mathbf{g}}_k^H \mathbf{p}_k\}}{\mu_{s,k}^m} - \frac{\parallel \hat{\mathbf{g}}_k^H \mathbf{p}_k^m \parallel^2 \mu_{s,k}}{(\mu_{s,k}^m)^2} - \geq \tau_{s,k}, \forall k \in K_2, \tag{28}$$

where $\mathcal{R}$ represents the real number. Finally, the optimization problem (13) at iteration m can be reformulated as

$$\max_{\mathbf{P},\mathbf{a},v,\boldsymbol{\tau},\boldsymbol{\tau_c},\boldsymbol{\tau_s},\boldsymbol{\mu},\boldsymbol{\mu_c},\boldsymbol{\mu_s}} \quad v \tag{29}$$
$$s.t. : (12b), (12e), (14), (15), (18), (19), (21), (22), (24), (26), (27), (28),$$

Problem (29) is now a convex problem that can be solved iteratively using success convex approximation (SCA). The details of this solution are illustrated in **Algorithm 1**. The algorithm works as follows. For given values of time slot allocation (α), and RIS phase shifts in the two time slots ($\boldsymbol{\Phi_{(1)}}, \boldsymbol{\Phi_{(2)}}$), the algorithm starts with assuming initial values for the variables $\mathbf{P}, \boldsymbol{\tau}, \boldsymbol{\tau_c}, \boldsymbol{\tau_s}, \boldsymbol{\mu}$, $\boldsymbol{\mu_c}$, and $\boldsymbol{\mu_s}$. These variables are used to solve (29). We get the updated values of the mentioned parameters and the objective function. The algorithm continues iteratively updating these variables till convergence. Then, we compare the new optimized Max-Min rate (objective function) with the previous value if it is greater than certain tolerance value (ϵ) this process will be repeated until convergence.

Algorithm 1: Joint precoding matrix and common rate allocation optimization algorithm

Given : α, $\boldsymbol{\Phi_{(1)}}$, $\boldsymbol{\Phi_{(2)}}$, and tolerance ϵ

Initialize: $\mathbf{P}^{(0)}, \boldsymbol{\tau}^{(0)}, \boldsymbol{\tau_c}^{(0)}, \boldsymbol{\tau_s}^{(0)}, \boldsymbol{\mu}^{(0)}, \boldsymbol{\mu_c}^{(0)}, \boldsymbol{\mu_s}^{(0)}, v^{(0)} = 0$;

```
m = 0;
Solve (29) using P^(0),τ^(0),τc^(0),τs^(0),μ^(0),μc^(0),μs^(0);
Find the optimal value for the objective function v*
  and optimal variables P^(*),τ^(*),τc^(*),τs^(*),μ^(*),μc^(*),
  μs^(*) ;
m = m + 1;
Update the old variables v^(1) ← v^(*), P^(1) ← P^(*),
  τ^(1) ← τ^(*), τc^(1) ← τc^(*), τs^(1) ← τs^(*),
  μ^(1) ← μ^(*), μc^(1) ← μc^(*), μs^(1) ← μs^(*);
while | v^(m) − v^(m−1) |> ε do
    m = m + 1;
    Solve (29) using P^(m−1),τ^(m−1),τc^(m−1),τs^(m−1),
      μ^(m−1),μc^(m−1),μs^(m−1);
    Find the optimal value for the objective function
      v* and optimal variables P^(*), τ^(*), τc^(*), τs^(*),
      μ^(*), μc^(*), μs^(*);
    Update the old variables: v^(m) ← v^(*),
      P^(m) ← P^(*), τ^(m) ← τ^(*), τc^(m) ← τc^(*),
      τs^(m) ← τs^(*), μ^(m) ← μ^(*), μc^(m) ← μc^(*),
      μs^(m) ← μs^(*);
end
```

B. Phase Shift Optimization in the First Time Slot

We now solve the optimization problem of RIS phase shifts during the direct transmission phase. The problem can be reformulated as follows. Given the time slot allocation, precoding matrix, and the common rate allocation, problem (12) can be reformulated as

$$\max_{\boldsymbol{\Phi_{(1)}},v} \quad v \tag{30}$$
$$s.t. : (12a), (12c), (12d), (13a), (13b), ,$$

We define $\Psi = [\psi_1, ..., \psi_N]^T$, where $\psi_n = e^{j\phi_n^{(1)}}, \forall n \in N$, and $R_0 = 2^{\frac{\sum_{i=1}^{K_1} a_i}{\alpha}} - 1$. Now, by changing the variables, we get $\mathbf{h}_k^H \boldsymbol{\Phi_{(1)}} \mathbf{G} \mathbf{p}_i = \mathbf{b}_{k,i}^H \Psi$, where $\mathbf{b}_{k,i} = (\text{diag}(\mathbf{h}_k^H)\mathbf{G}\mathbf{p}_i)^*$. To further simplify the expression, we define $g_{k,i} = \mathbf{g}_k^H \mathbf{p}_i$ Furthermore, we introduce slack variables $\boldsymbol{\sigma} = [\sigma_1, ..., \sigma_{K_1}]$, and $\boldsymbol{\gamma} = [\gamma_1, ..., \gamma_{K_1}]$, where $\boldsymbol{\sigma}$ and $\boldsymbol{\gamma}$ represent the SINR and the rate vectors of private streams of users in group I respectively. We also define $\boldsymbol{\sigma_c} = [\sigma_{c,1}, ..., \sigma_{K_1}]$, and $\boldsymbol{\gamma_c} = [\gamma_{c,1}, ..., \gamma_{c,K_1}]$, for $k \in K_1$ and $k \neq A$, where $\boldsymbol{\sigma_c}$ and $\boldsymbol{\gamma_c}$ represent the SINR and the rate vectors of common streams of users in group I respectively. While, $\boldsymbol{\sigma_s} = [\sigma_{s,1}, ..., \sigma_{s,K_2}]$, and $\boldsymbol{\gamma_s} = [\gamma_{s,1}, ..., \gamma_{s,K_2}]$, where $\boldsymbol{\sigma_s}$ and $\boldsymbol{\gamma_s}$ represent the SINR and the rate vectors of private streams of users in group II respectively.

$$\max_{\Psi,\boldsymbol{\sigma},\boldsymbol{\sigma_c},\boldsymbol{\sigma_s},\boldsymbol{\gamma},\boldsymbol{\gamma_c},\boldsymbol{\gamma_s},v} \quad v \tag{31}$$

$$s.t. : \alpha\gamma_k + a_k \geq v, \forall k \in K_1, \tag{31a}$$

$$\alpha\gamma_{s,k} \geq v, \forall k \in K_2, \tag{31b}$$

$$\alpha\gamma_{c,k} + c_k^{(2)} \geq \sum_{i=1}^{K_1} a_i, k \in K_1, k \neq A, \tag{31c}$$

$$1 + \sigma_k - 2^{\gamma_k} \geq 0, \forall k \in K_1, \tag{31d}$$

$$1 + \sigma_{c,k} - 2^{\gamma_{c,k}} \geq 0, k \in K_1, k \neq A, \tag{31e}$$

$$1 + \sigma_{s,k} - 2^{\gamma_{s,k}} \geq 0, \forall k \in K_2, \tag{31f}$$

$$\mid \psi_n \mid = 1, \forall n \in N, \tag{31g}$$

$$\frac{\parallel \mathbf{b}_{k,k}^H \Psi + g_{k,k} \parallel^2}{\sum_{\substack{i=1 \\ i \neq k}}^{K} \parallel \mathbf{b}_{k,i}^H \Psi + g_{k,i} \parallel^2 + N_0} \geq \sigma_k, \forall k \in K_1, \tag{31h}$$

$$\frac{\parallel \mathbf{b}_{k,0}^H \Psi + g_{k,0} \parallel^2}{\sum_{i=1}^{K} \parallel \mathbf{b}_{k,i}^H \Psi + g_{k,i} \parallel^2 + N_0} \geq \sigma_{c,k}, k \in K_1, k \neq A, \tag{31i}$$

$$\frac{\parallel \mathbf{b}_{k,k}^H \Psi + g_{k,k} \parallel^2}{\sum_{\substack{i=0 \\ i \neq k}}^{K} \parallel \mathbf{b}_{k,i}^H \Psi + g_{k,i} \parallel^2 + N_0} \geq \sigma_{s,k}, \forall k \in K_2, \tag{31j}$$

$$\frac{\parallel \mathbf{b}_{A,0}^H \Psi + g_{A,0} \parallel^2}{\sum_{i=1}^{K} \parallel \mathbf{b}_{A,i}^H \Psi + g_{A,i} \parallel^2 + N_0} \geq R_0, \tag{31k}$$

Since (31g) is non-convex, then penalty method is adapted and hence problem (31) can be rewritten as follows

$$\max_{\Psi, \boldsymbol{\sigma}, \boldsymbol{\sigma_c}, \boldsymbol{\sigma_s}, \boldsymbol{\gamma}, \boldsymbol{\gamma_c}, \boldsymbol{\gamma_s}, v} \quad v + Z \sum_{n=1}^{N} (\mid \psi_n \mid^2 - 1), \tag{32}$$

$$s.t. : \mid \psi_n \mid \leq 1, \forall n \in N, \tag{32a}$$

$$(31a) - (31f), (31h) - (31k),$$

where Z is a large positive constant value. Also, (31h), (31i) and (31j) are non-convex, therefore, we introduce slack variables to handle their non-convexity $\boldsymbol{\beta}$= $[\beta_1, .., \beta_{K_1}]$, and $\boldsymbol{\beta_c}$= $[\beta_{c,1}, .., \beta_{c,K_1}]$, for $k \in K_1$ and $k \neq A$, which are interference-to-noise for private and common streams of users in group I respectively. We also define $\boldsymbol{\beta_s}$= $[\beta_{s,1}, .., \beta_{s,K_2}]$, which is interference-to-noise for private of users in group II respectively. Then (31h) can be rewritten as follows

$$\begin{aligned} \parallel \mathbf{b}_{k,k}^H \Psi + g_{k,k} \parallel^2 &\geq \sigma_k \beta_k, \forall k \in K_1, \\ &= \frac{1}{4}((\sigma_k + \beta_k)^2 - (\sigma_k - \beta_k)^2). \end{aligned} \tag{33}$$

where β_k is the denominator of SINR equation in (31h), thus its expression is given by

$$\sum_{\substack{i=1 \\ i \neq k}}^{K} \parallel \mathbf{b}_{k,i}^H \Psi + g_{k,i} \parallel^2 + N_0 \leq \beta_k, \forall k \in K_1, \tag{34}$$

in the same way as (31h), (31i) can be rewritten as

$$\begin{aligned} \parallel \mathbf{b}_{k,0}^H \Psi + g_{k,0} \parallel^2 &\geq \sigma_{c,k} \beta_{c,k}, k \in K_1, k \neq A, \\ &= \frac{1}{4}((\sigma_{c,k} + \beta_{c,k})^2 - (\sigma_{c,k} - \beta_{c,k})^2). \end{aligned} \tag{35}$$

$$\sum_{i=1}^{K} \parallel \mathbf{b}_{k,i}^H \Psi + g_{k,i} \parallel^2 + N_0 \leq \beta_{c,k}, k \in K_1, k \neq A, \tag{36}$$

Similarly (31j) can also be written as

$$\begin{aligned} \parallel \mathbf{b}_{k,k}^H \Psi + g_{k,k} \parallel^2 &\geq \sigma_{s,k} \beta_{s,k}, \forall k \in K_2 \\ &= \frac{1}{4}((\sigma_{s,k} + \beta_{s,k})^2 - (\sigma_{s,k} - \beta_{s,k})^2). \end{aligned} \tag{37}$$

$$\sum_{\substack{i=0 \\ i \neq k}}^{K} \parallel \mathbf{b}_{k,i}^H \Psi + g_{k,i} \parallel^2 + N_0 \leq \beta_{s,k}, \forall k \in K_2, \tag{38}$$

Finally, (31k) can be rewritten as

$$\parallel \mathbf{b}_{A,0}^H \Psi + g_{A,0} \parallel^2 \geq R_0 \sum_{i=1}^{K} \parallel \mathbf{b}_{A,i}^H \Psi + g_{A,i} \parallel^2 + N_0, \tag{39}$$

Using all the above transformations, problem (31) can be reformulated as

$$\max_{\substack{\Psi, \boldsymbol{\sigma}, \boldsymbol{\sigma_c}, \boldsymbol{\sigma_s}, \\ \boldsymbol{\gamma}, \boldsymbol{\gamma_c}, \boldsymbol{\gamma_S}, \boldsymbol{\beta_c}, \boldsymbol{\beta_s}, v}} \quad v + Z \sum_{n=1}^{N} (\mid \psi_n \mid^2 - 1), \tag{40}$$

$$s.t. : (31a) - (31f), (32a), (33) - (39),$$

Now, problem (40) can be solved using SCA as described in **Algorithm 2**. We used the optimized value of **P** and **a** from **Algorithm 1**. For given value of time slot allocation (α), we start by initializing RIS phase shift in first time slot ($\Psi^{(0)}$), then we solve (40) using these initial values. Then after solving (40), the new optimized values for the Max-Min rate (v^*) and (Ψ^*) are obtained. Then, the the Max-Min rate ($v^{(m)}$) and ($\Psi^{(m)}$) are updated to the new optimized values. Then, the new optimized Max-Min rate is compared with the previous value; if it is greater than certain tolerance value (ϵ), the process is repeated until convergence. The final optimized RIS phase shifts is the diagonal of the last vector obtained for (Ψ^*) inside the while loop.

Algorithm 2: Phase shift optimization in the first time slot

Given : α, **P**, **a**, $\boldsymbol{\Phi_{(2)}}$, and tolerance ϵ
Initialize: $\Psi^{(0)}$, $v^{(0)} = 0$;

$m = 0$;
Solve (40) using $\Psi^{(0)}$;
Find the optimal value for the objective function $v^{(*)}$ and optimal value of $\Psi^{(*)}$;
$m = m + 1$;
Update the old variables $v^{(1)} \leftarrow v^{(*)}$, $\Psi^{(1)} \leftarrow \Psi^*_{(1)}$;
while $\mid v^{(m)} - v^{(m-1)} \mid > \epsilon$ **do**
  $m = m + 1$;
  Solve (40) using $\Psi^{(m-1)}$;
  Find the optimal value for the objective function $v^{(*)}$ and optimal value of $\Psi^{(*)}$;
  Update the old variables $v^m \leftarrow v^*$, $\Psi^{(m)} \leftarrow \Psi^{(*)}$;
end
$\Phi_{(1)} = \text{diag } (\Psi^{(m)})$;

C. Phase Shift Optimization in the Second Time Slot

The last part of optimization is to optimize the RIS phase shifts in cooperative transmission phase. So given the optimized time slot allocation, precoding matrix, common rate allocation and phase shifts of RIS in the first time slot, problem (12) can be reformulated as

$$\max_{\Phi_{(2)},v} \quad v \tag{41}$$
$$s.t. : (12a), (12c), (12d), (13a), ,$$

We define $\boldsymbol{\omega} = [\omega_1, ..., \omega_N]^T$, where $\omega_n = e^{j\phi_n^{(2)}}, \forall n \in N$. We will change the channel variables to be in term of $\boldsymbol{\omega}$, thus $\mathbf{h}_{k,r}^H \boldsymbol{\Phi}_{(2)} \mathbf{h}_{A,r} = \mathbf{d}_{k,A}^H \boldsymbol{\omega}$, where $\mathbf{d}_{k,A} = (\text{diag}(\mathbf{h}_{k,r}^H)\mathbf{h}_{A,r})^*$. Furthermore, we introduced slack variable $\boldsymbol{\lambda} = [\lambda_1, ..., \lambda_{K_1}]$, for $k \in K_1$ and $k \neq A$, where $\boldsymbol{\lambda}$ is rate of common stream of users in group I in the second time slot. Hence, problem (41) can be reformulated as

$$\max_{\boldsymbol{\omega},\boldsymbol{\lambda},v} \quad v \tag{42}$$
$$s.t. : r_k^{(1)} + a_k \geq v, \forall k \in K_1, \tag{42a}$$
$$c_k^{(1)} + \lambda_k \geq \sum_{i=1}^{K_1} a_k, k \in K_1, k \neq A, \tag{42b}$$
$$| \omega_n | = 1, \forall n \in N, \tag{42c}$$
$$(1-\alpha)\log_2(1 + \frac{P_r \parallel \mathbf{d}_{k,A}^H \boldsymbol{\omega} + h_{A,k} \parallel^2}{N_0}) \geq \lambda_k, \quad k \in K_1, k \neq A, \tag{42d}$$

where $r_k^{(1)}$ and $c_k^{(1)}$ both can be calculated, as all their variables is now optimized. Since (42c) is non-convex, then penalty method is adapted and hence problem (42) can be rewritten as follows

$$\max_{\boldsymbol{\omega},\boldsymbol{\lambda},v} \quad v + T\sum_{n=1}^{N}(| \omega_n |^2 - 1), \tag{43}$$
$$s.t. : | \omega_n | \leq 1, \forall n \in N, \tag{43a}$$
$$(42a), (42b), (42d),$$

where T is a large positive constant value. Then problem (43) can be solved using SCA method, which is explained in **Algorithm 3**. In **Algorithm 3**, we use the optimized value of **P** and **a** from **Algorithm 1**, and the optimized RIS phase shifts in the first time slot ($\boldsymbol{\Phi}_{(1)}$) from **Algorithm 2**. For given value of time slot allocation (α), we start by initializing RIS phase shift in the second time slot ($\boldsymbol{\omega}^{(0)}$), then we solve (43) using these initial values. Then after solving (43), the new optimized values for the Max-Min rate (v^*) and ($\boldsymbol{\omega}^*$) are obtained. Then, the Max-Min rate (v^m) and ($\boldsymbol{\omega}^m$) are updated to the new optimized values. Then, the new optimized Max-Min rate is compared with the previous value; if it is greater than certain tolerance value (ϵ), the process is repeated until convergence. The final optimized RIS phase shifts is the diagonal of the last vector obtained for ($\boldsymbol{\omega}^*$) inside the while loop.

Algorithm 3: Phase shift optimization in the second time slot

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Given : optimized α, P, **a**, $\boldsymbol{\Phi}_{(1)}$, and tolerance ϵ;
Calculate: $r_k^{(1)}, \forall k \in K$ and $c_k^{(1)}$, for $k \in K_1, k \neq A$;
Initialize : $= \boldsymbol{\omega}^{(0)}$, $v^{(0)} = 0$;

$m = 0$;
Solve (43) using $\boldsymbol{\omega}^{(0)}$;
Find the optimal value for the objective function $v^{(*)}$ and optimal value of $\boldsymbol{\omega}^{(*)}$;
$m = m + 1$;
Update the old variables $v^{(1)} \leftarrow v^*$, $\boldsymbol{\omega}^{(m)} \leftarrow \boldsymbol{\omega}^{(*)}$;
while $| v^{(m)} - v^{(m-1)} | > \epsilon$ **do**
  $m = m + 1$;
  Solve (43) using $\boldsymbol{\omega}^{(m-1)}$;
  Find the optimal value for the objective function v^* and optimal value of $\boldsymbol{\omega}^{(*)}$;
  Update the old variables $v^m \leftarrow v^*$, $\boldsymbol{\omega}^{(m)} \leftarrow \boldsymbol{\omega}^{(*)}$;
end
$\Phi_{(2)} = \text{diag }(\boldsymbol{\omega}^{\boldsymbol{m}})$;

D. Overall Optimization

The detailed explanation of the optimization technique after combing all the sub-problems is explained in this section. We optimize the time slot allocation α using exhaustive search as mentioned before. So for each value of α, the three sub-problems are solved as follows: A random RIS phase shifts for the first and second time slot ($\boldsymbol{\Phi}_{(1)}, \boldsymbol{\Phi}_{(2)}$) are used to optimize the precoding matrix (**P**) and the common rate allocation (**a**) according to **Algorithm 1**, this algorithm obtains optimized values for (**P**) and (**a**). These optimized values are fed into **Algorithm 2** in order to optimize the phase shifts of RIS in the first time slot ($\boldsymbol{\Phi}_{(1)}$). The optimized parameters from **Algorithm 1** and **Algorithm 2** are used in **Algorithm 3** to optimize the phase shifts of RIS in the second time slot ($\boldsymbol{\Phi}_{(2)}$) and to obtain the optimized Max-Min rate. Then, the last optimized minimum rate is compered with previous value obtained and if the difference between them is greater than the tolerance (ϵ), these steps are repeated again until the Max-Min rate is converged. Then procedure is repeated for each value of α. Finally, the optimized time slot is the one that corresponds to the maximum value of Max-Min rate, and is used to calculate the optimized minimum rate of all users. This optimization technique is called alternative optimization (AO) which can be solved using YALMIP optimization tool. The procedure of the overall optimization is illustrated in **Algorithm 4**.

VI. Numerical Results

Numerical results are obtained to evaluate the performance of selective multi-user cooperative rate splitting assisted by RIS. To illustrate the superiority of selective CRS-RIS, comparison with the following systems is performed:

Algorithm 4: Alternative optimization algorithm

Given : tolerance ϵ
Initialize : $\mathbf{\Phi}^{(\mathbf{0})}_{(\mathbf{1})}, \mathbf{\Phi}^{(\mathbf{0})}_{(\mathbf{2})}, v^{(0)} = 0$;

counter=1;
for $\alpha = 0.1 : 0.1 : 0.9$ **do**
  $m = 0$;
  Given $\mathbf{\Phi}^{(\mathbf{0})}_{(\mathbf{1})}$ and $\mathbf{\Phi}^{(\mathbf{0})}_{(\mathbf{2})}$, use **Algorithm 1** to optimize the precoding matrix ($\mathbf{P}$), common rate allocation ($\mathbf{a}$);
  $m = m + 1$;
  The optimized solution is donated by $(\mathbf{P}^{(1)}, \mathbf{a}^{(1)})$;
  Given $(\mathbf{P}^{(1)}, \mathbf{a}^{(1)})$, use **Algorithm 2** to optimize the RIS phase shifts in the first time slot $(\mathbf{\Phi}_{(\mathbf{1})})$, and the solution is donated by $(\mathbf{\Phi}^{(\mathbf{1})}_{(\mathbf{1})})$;
  Given $(\mathbf{P}^{(1)}, \mathbf{a}^{(1)}, \mathbf{\Phi}^{(\mathbf{1})}_{(\mathbf{1})})$, use **Algorithm 3** to optimize the RIS phase shifts in the second time slot $(\mathbf{\Phi}_{(\mathbf{2})})$, and the solution is donated by $(\mathbf{\Phi}^{(\mathbf{1})}_{(\mathbf{2})})$ and donate the objective value by $v^{(1)}$;
  while $\mid v^{(m)} - v^{(m-1)} \mid > \epsilon$ **do**
    $m = m + 1$;
    Given $\mathbf{\Phi}^{(\mathbf{m-1})}_{(\mathbf{1})}$ and $\mathbf{\Phi}^{(\mathbf{m-1})}_{(\mathbf{2})}$, use **Algorithm 1** to optimize the precoding matrix ($\mathbf{P}$), common rate allocation ($\mathbf{a}$), and the optimized solution is donated by $(\mathbf{P}^{(m)}, \mathbf{a}^{(m)})$;
    Given $(\mathbf{P}^{(m)}, \mathbf{a}^{(m)})$, use **Algorithm 2** to optimize the RIS phase shifts in the first time slot $(\mathbf{\Phi}_{(\mathbf{1})})$, and the solution is donated by $(\mathbf{\Phi}^{(\mathbf{m})}_{(\mathbf{1})})$;
    Given $(\mathbf{P}^{(m)}, \mathbf{a}^{(m)}, \mathbf{\Phi}^{(\mathbf{m})}_{(\mathbf{1})})$, use **Algorithm 3** to optimize the RIS phase shifts in the second time slot $(\mathbf{\Phi}_{(\mathbf{2})})$, and the solution is donated by $(\mathbf{\Phi}^{(\mathbf{m})}_{(\mathbf{2})})$ and donate the objective value by $v^{(m)}$;
  end
  $\mathbf{q}$ (counter) = $v^{(m)}$;
  counter=counter+1;
end
v = max($\mathbf{q}$) ;
Get the position of the maximum v in $\mathbf{q}$;
Get the value of α at this position, and this will be the optimized time slot allocation;

- **Selective RSMA RIS:** The proposed system in Section III.
- **CRS RIS:** Cooperative RSMA assisted by RIS.
- **RSMA RIS:** RSMA assisted RIS without user cooperation.
- **NO RIS CRS:** Cooperative rate splitting without usage of RIS.
- **NO RIS RSMA:** Rate splitting without user relaying and without usage of RIS.

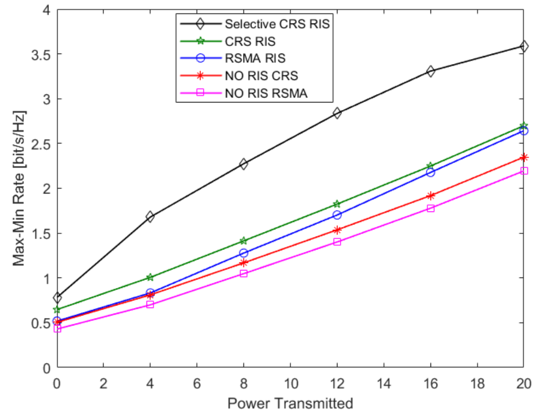

Fig. 2: Max-Min rate vs. power transmitted

In our simulation, user 1 will be the user relay ($U_A = U_1$), as it is the one with best channel condition. Also, user 4 is chosen to not apply RSMA principle by treating the common stream as noise, this is because it has the worst channel condition since it is the farthest one from the BS. As mentioned earlier, the selection of users' group is based on the channel condition to solve the RSMA limitation and this what we have done here. we assume that $P_t = P_r$. The simulation parameters are given in Table I.

Fig. 2 shows the Max-Min rate of different systems mentioned above versus the transmitted power. It is shown that the Max-Min rate of the users increases as the power increases. Also, it is noticeable that the selective CRS-RIS has the highest rate when compered to other systems. The Max-Min rate increases by 55.48% using selective CRS-RIS compared with CRS-RIS at 12dB. This shows the superiority of the selective CRS-RIS over all systems. The figure also shows that, the systems RSMA-RIS and NO-RIS-CRS have better performance than NO-RIS-RSMA which illustrates that integration of RSMA with cooperative relaying and RIS improves the system performance. The figure also shows that systems that use RSMA has better performance than other systems. This is because RSMA manages the interference by partially decoding the interference and partially treating it as noise.

TABLE I: Simulation parameters

Parameter	Value
BS antenna, N_t	2
Power transmitted, P_t	12 dB
AWGN power, N_0	-40 dB
Path-loss exponent	2
Tolerance, ϵ	0.1
Number of users, K_1	3
Number of users, K_2	1
Number of reflecting elements, N	4
Base station (BS) position	(0,0)
RIS position	(40,10)
First user, U1	(40,0)
Second user, U2	(50,0)
Third user, U3	(60,0)
Fourth user, U4	(80,0)

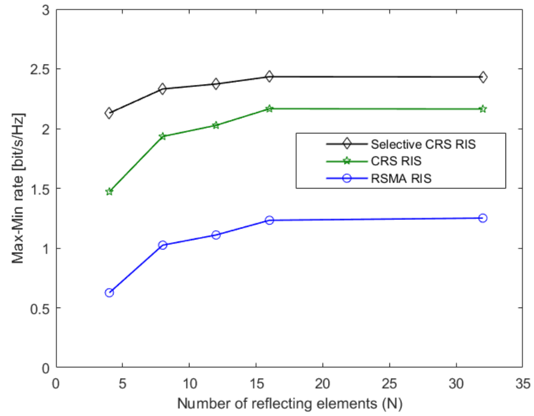

Fig. 3: Max-Min rate vs. number of reflecting elements

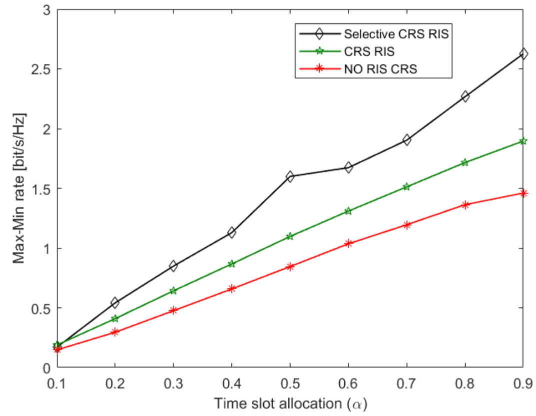

Fig. 5: Max-Min rate vs. time slot allocation

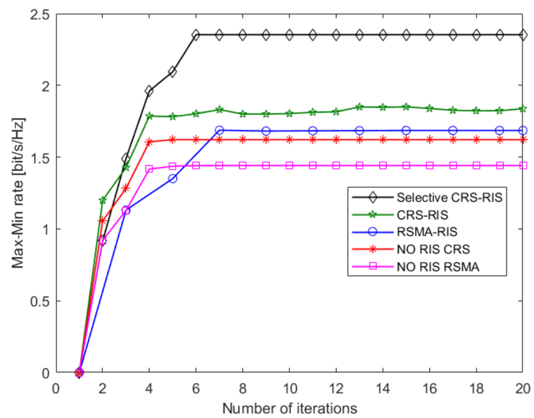

Fig. 4: Max-Min rate vs. number of iterations

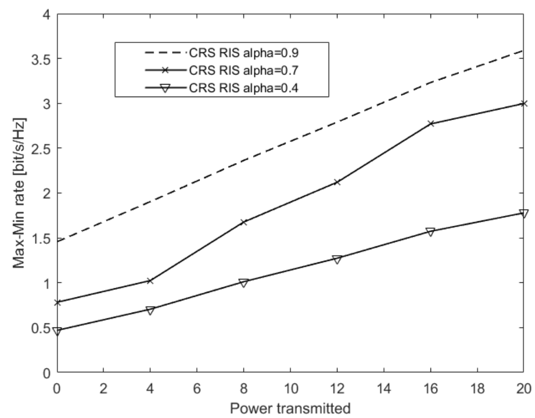

Fig. 6: Max-Min rate vs. power transmitted

Fig. 3 shows the Max-Min rate versus number of reflecting elements (N) of the RIS. The figure is plotted with P_t = 10 dB. The figure shows that as the number of reflecting elements increases, the Max-Min rate increases for all systems. This is because by increasing the number of reflecting elements, the directivity and the reflected power increased towards the intended user. The figure also shows that when the number of the reflecting elements tends to large number, the Max-Min rate tends to a constant value. This is due to the increase of the interference. It also shown that selective CRS-RIS outperforms CRS-RIS by 44.7% at N=4, and outperforms RMSA-RIS by 240.2%.

Fig. 4 shows the convergence of the AO algorithm. It plots the Max-Min rate versus the number of iterations. It can be seen that all the strategies converges within the first 10 iterations.

Fig. 5 shows the Max-Min rate versus the time slot allocation (α). It shows that as the time slot allocation (α) increases the rate increases as well. The reason behind that is as α increases, the resource allocated to the direct transmission phase increases and hence the rate of direct transmission increases. Since, the direct transmission phase includes both the the common and private streams, then as the rate of direct transmission increases the rate of both private and common streams increases as well which increases the Max-Min rate. Note that, the cooperative transmission phase has a weaker effect on the Max-Min rate because it includes only the rate of the common stream. As shown before, the two systems selective CRS-RIS and CRS-RIS overcome the system NO-RIS-CRS due to the existence of RIS.

Fig. 6 shows the Max-Min rate for different time slot allocation (α) versus the transmitted power for the selective CRS-RIS system. The figure shows that as the transmitted power increases, the Max-Min rate increases. The results also show that as time slot allocation increases the rate increases due to increasing the resources allocated to the direct transmission phase.

To check different selection configurations, Fig. 7 is presented for this purpose. Fig. 7 shows Max-Min rate versus the transmitted power. The figure compares between two selective CRS-RIS systems, one when user-4 is selected as the user that does not apply RSMA principle, while the other when user-3 is selected instead. It is noted that, user-4 has the worst channel gain than user-3, since it is the farthest user from the BS. The

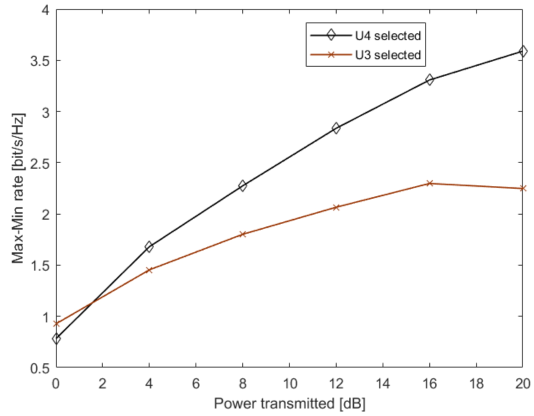

Fig. 7: Max-Min rate vs. power transmitted for different selected user

system with the selection of user-4 has better performance than the system with the selection of user-3. The result of Fig. 7 confirms the idea of the proposed selective CRS-RIS that select the user with the worst channel gain. This criteria enhances the system performance.

VII. Conclusion

In this paper, a multi-user selective CRS assisted by RIS for downlink MISO transmission network has been proposed. This system selects the users with the worst channel conditions to not perform RSMA, while the other users perform RMSA normally. The reason of this approach is to overcome the drawback of the limitation of system performance due to the low rate of common streams of the weak channel users. The objective is to maximize the minimum rate of the users. Hence, alternative optimization is used to optimize time slot allocation, precoding matrix, common rate allocation and phase shifts of RIS in the two time slots. Numerical results ensure the superiority of proposed selective CRS-RIS over the other systems, where the Max-Min rate increases by 55.48 % over non selective CRS-RIS. As for the future work, the proposed system can be extended to use 2-layer RSMA principle, and compare its performance with 1-layer RSMA. Also, the investigation of the performance of proposed system in case of imperfect CSIT is a controversial topic. Furthermore, Active and hybrid RIS can be used instead of passive RIS, to study the effect of RIS type on the system performance.

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